Torsion driven inflationary magnetogenesis

Rahul Kothari,[1] M. V. S. Saketh,[2] and Pankaj Jain[3]

[1]Department of Physics and Astronomy, The University of the Western Cape, Bellville 7535, South Africa
[2]Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA
[3]Department of Physics, Indian Institute of Technology, Kanpur 208016, India

(Received 12 March 2020; accepted 11 June 2020; published 1 July 2020)

We show that the breaking of the conformal invariance of the electromagnetic Lagrangian, which is required for inflationary magnetogenesis, arises naturally in the Poincaré gauge theory. We use the minimal coupling prescription to introduce the electromagnetic gauge fields as well as non-Abelian gauge fields in this theory. Due to the addition of non-Abelian gauge fields, we show that the solar constraints on this model can be naturally evaded. We find that in the minimal version of this model the generated magnetic field is too small to explain the observations. We discuss some generalizations of the gravitational action, including the Starobinsky model and a model with conformal invariance. We show that such generalizations naturally generate the kinetic energy terms required for magnetogenesis. We propose a generalization of the minimal model by adding a potential term, which is allowed within the framework of this model, and show that it leads to sufficiently large magnetic fields.

DOI: 10.1103/PhysRevD.102.024008

I. INTRODUCTION

Magnetic fields are found in almost all bound structures, such as galaxies, stars, star clusters, and even some planets like Earth [1–3]. Recent observational evidence [4–7] suggests that even cosmic voids contain magnetic fields of order 10^{-16} G with correlation lengths of 1 Mpc or more. The upper limit on the magnetic field on such scales is of order 10^{-9} G [8]. Such large-scale correlation is hard to explain using astrophysical processes. This is because one would expect astrophysical processes to generate these fields during the radiation-dominated phase, and the small Hubble radius during this epoch cannot account for such large-scale correlations. Even large-scale magnetic fields in galaxies [9] and clusters of galaxies in the distant past are not easy to explain. One possibility is that these fields are actually a relic from the inflationary era [10–12]. This explains why they are supposedly present in the voids. The same mechanism also explains the origin of strong magnetic fields in bound structures by serving as the seed field that was amplified via a dynamo mechanism [13] in these structures.

There exist many models of inflationary magnetogenesis in which the magnetic field is amplified many folds during inflation [10,14–22]. All of these models break the conformal invariance of the electromagnetic Lagrangian, which is required to amplify the vacuum fluctuations of electromagnetic fields. In the model developed by Ratra [14], the electromagnetic Lagrangian is coupled with the inflaton field to break the conformal invariance. It turns out that for some parameter values this model suffers from the strong coupling or backreaction problem [23,24]. There exist several proposals [25–27] to generate cosmic magnetic fields of the required strength without facing either of these problems, if conformal invariance is broken during inflation. However, there also exist additional constraints from the cosmic microwave background which impose further limits on the magnetic field that can be generated [28–30].

Although there exist several proposals [10,15–22] which break conformal invariance, these do not explain the coupling between electromagnetic and scalar fields assumed in the Ratra model [14]. So far, this coupling has been put in by hand. The only exception is the string-inspired model [16] which has some similarity to the Ratra model. We propose that such a coupling naturally arises in Poincaré gauge theory [31], when torsion—a dynamical variable of the theory—is taken into account. It turns out that for a specific choice of torsion, sourced by a scalar field ϕ—referred to as a "tlaplon" in the literature [32]—one can create a scenario where both inflation and magnetogenesis is driven by the tlaplon itself. In this formalism a restricted version of torsion is coupled directly to the electromagnetic field in a gauge-invariant manner. It has been argued in the literature [31,33,34] that an alternative procedure is to not couple torsion to photons. As mentioned in Ref. [31], this has the strange consequence that for the photon field the spin tensor vanishes, whereas this would not be the case for a massive vector field. This has been justified by the argument that the photon is special since it is massless [33,34]. However, this may lead to inconsistencies. A particular problem that we discuss in this paper is the implications of this procedure if the mass of the vector field is generated by the Higgs mechanism. In this case, we would not couple the gauge field to torsion. However, the gauge field becomes massive due to the Higgs mechanism and this massive field would then not couple to torsion either. This is inconsistent since a massive field with spin must couple to torsion.

There also exists considerable literature [35–41] regarding attempts to derive the coupling of the gauge fields to torsion by dimensional reduction of a five-dimensional theory using the Kaluza-Klein formalism. This has the advantage of preserving both Poincaré gauge invariance and electromagnetic gauge invariance in four dimensions, and hence can settle the question of how the electromagnetic field should be introduced in such a theory. In particular, it was argued in Ref. [36] that under certain conditions the formalism proposed in Ref. [32] can be derived from the five-dimensional theory. Essentially, Ref. [36] assumed a limited form of torsion in five dimensions and expressed it in terms of a scalar field Ω. This is analogous to the procedure used in Ref. [32] in four dimensions. Furthermore, it was shown that the field Ω, and hence the torsion potential ϕ, is related to the g_{55} component of the metric. However, this was shown to be invalid in Refs. [38,39] and hence this formalism does not provide guidance of how to couple the electromagnetic field to torsion. A more detailed analysis of the five-dimensional theory was provided in Ref. [40]. In that paper, the authors did not use any specific form of torsion and used the anholonomic horizontal lift basis for reduction to four dimensions. The authors found that the electromagnetic field decouples from torsion. This is easily understood. In Ref. [40] the authors used only one term in the gravitational action, i.e., the five-dimensional analogue of the Ricci scalar $\hat{R}$, including the contribution due to torsion (Eq. 2.6 of Ref. [40]). This term does not generate any derivatives of the torsion field. Hence, starting from the five-dimensional action we do not expect any terms involving derivatives of the electromagnetic field A_μ in the terms associated with torsion. Since a term independent of derivatives of A_μ cannot be gauge invariant, we do not expect such terms in the action. Although this is the standard form of the action, other terms, such as $\hat{R}^2$, can also included [42]. With these terms we do, in general, find a coupling between torsion and the electromagnetic field. Although this does not provide a derivation of the coupling proposed in Ref. [32], it does suggest that a coupling should exist. The Hojman *et al.* [32] procedure may be regarded as an effective simple framework to implement such a coupling.

The article is structured in the following manner. In Sec. II, we discuss in brief the possibility of a primordial origin of magnetic fields, Ratra's model [14], and its drawbacks. This is followed by a brief introduction to the Poincaré gauge theory of gravity in Sec. III. It is found that torsion (a dynamical variable in Poincaré gauge theory) generically coupled to the electromagnetic field breaks gauge invariance. But this can be avoided by choosing a restricted form of torsion, as has been suggested in Ref. [32]. This choice of torsion leads to some problems, as has been pointed out in Ref. [43]. In later sections we see how these and similar problems can be evaded. Finally, in Sec. VI we see how magnetogenesis can be brought about by assuming the restricted form.

II. PRIMORDIAL ORIGIN OF MAGNETIC FIELDS

The basic idea is that the magnetic fields are a relic of inflation. This means that the vacuum fluctuations of the electromagnetic field get amplified during inflation and subsequently become classical fluctuations in the later phases of the evolution of the Universe. This inflationary paradigm also accounts for the origin of classical density perturbations [44]. However, it is known that the electromagnetic Lagrangian is conformally invariant, which is tantamount to saying that the equations of motion for the fields are not modified by curvature induced due to a metric that is conformally related to the Minkowski metric. Since the Friedmann-Robertson-Walker metric is conformally related to the Minkowski metric, no amplification of vacuum fluctuations can take place in this case. In fact, due to spatial expansion, the energy density of the electromagnetic field varies as a^{-4}. Hence, one needs to break the conformal invariance of electromagnetism if one wants to bring about any amplification.

In Ref. [14] Ratra proposed a coupling of the form

$$\sqrt{-g}e^{2\alpha\phi}F^{\mu\nu}F_{\mu\nu} \tag{2.1}$$

between the inflaton field (ϕ) and electromagnetic field $F_{\mu\nu}$ with α as a free parameter. He further showed that under slow-roll inflation conditions, this causes amplification of fields. However, it appears to pose some problems. For slow-roll inflation, let us assume that $e^{\alpha\phi}$ evolves as a^β. It turns out that a sufficient amount of scale-invariant amplification is achieved for some values of β [25–27]. However, in this case either the electronic charge becomes very large at the beginning of inflation or the electric field grows too fast and backreacts. Hence, we either end up facing the strong coupling or backreaction problem if we demand sufficient amplification for the magnetic field. A nice description of both of these problems can be found in Ref. [25]. In an alternate treatment of this problem, however, it is possible to get sufficiently strong magnetic fields with no backreaction [26,27]. In our analysis we shall follow the formalism developed in these papers.

III. A BRIEF REVIEW OF POINCARÉ GAUGE THEORY

The three fundamental interactions—weak, electromagnetic, and strong—are described within the framework of relativistic quantum field theories on flat Minkowski spacetime. These quantum fields reside in the spacetime but do not couple to it [34]. Moreover, all of these theories are gauge theories. Gravity seems to be different from these in that (a) it is a classical theory and (b) it is based on the deformation of spacetime itself. Upon quantization, one faces the problem of nonrenormalizability [45,46]. It is suggested that gauging, in addition to providing many other features, might provide a renormalizable version of gravity [47]. In addition to these reasons, it is natural to inquire whether gravitation can also be based on local gauge invariance [31,46]. Furthermore, it has been shown in the singularity theorems of Hawking and Penrose that a cosmology based on Riemannian geometry would force the Universe to either fall into or come out of a singularity. The simplest way of avoiding such consequences is to assume non-Riemannian geometries [47,48]. Poincaré gauge theory based on the local gauge invariance of the Poincaré group provides one such paradigm.

Mathematically speaking, the Poincaré group is a semidirect product [49] of the spacetime translations and the Lorentz group, i.e., $P(1,3) \equiv SO(1,3) \rtimes T(1,3)$ [50]. When we gauge this group, we find that instead of one, two gauge field multiplets are obtained. Historically, Utiyama was the first to apply gauge principles to generate gravitational interactions [51] by gauging the Lorentz group. Later, Kibble gauged the whole Poincaré group [31]. Here we must emphasize that different spacetime geometries are obtained upon gauging different groups. For example, if one considers only the translational group then one obtains Weitzenböck geometry, a geometry with torsion but no curvature [52]. The Poincaré gauge theory also assumes the metricity condition:

$$\nabla_\mu g_{\rho\sigma} = \partial_\mu g_{\rho\sigma} - \Gamma^\tau{}_{\mu\rho} g_{\tau\sigma} - \Gamma^\tau{}_{\mu\sigma} g_{\rho\tau} = 0, \tag{3.1}$$

where the covariant derivative is taken with respect to the affine connection $\Gamma^\alpha_{\beta\gamma}$. We point out that there are more general classes of theories known as metric affine theories where even the metricity condition [i.e., Eq. (3.1)] is not assumed. The reader is referred to Refs. [47,53] for more details.

According to general relativity, Minkowskian spacetime in the presence of matter becomes Riemannian. The geometry of a manifold is encoded in the connection $\Gamma^\alpha_{\beta\gamma}$ which in general can be written as [54]

$$\Gamma^\alpha_{\beta\gamma} = \overset{\circ}{\Gamma}{}^\alpha_{\beta\gamma} + K^\alpha_{\beta\gamma}, \tag{3.2}$$

where the quantity $K^\alpha_{\beta\gamma}$ is called the contortion and $\overset{\circ}{\Gamma}{}^\alpha_{\beta\gamma} = \overset{\circ}{\Gamma}{}^\alpha_{\gamma\beta}$ is the usual Christoffel symbol. Furthermore, the quantity

$$T^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta} \tag{3.3}$$

is called the torsion, which is clearly antisymmetric with respect to its last two indices. In addition to the condition (3.1), we also assume $\overset{\circ}{\nabla}_\mu g_{\rho\sigma} = 0$. This, together with $\overset{\circ}{\Gamma}{}^\alpha_{\beta\gamma} = \overset{\circ}{\Gamma}{}^\alpha_{\gamma\beta}$, allows us to solve for $\overset{\circ}{\Gamma}{}^\alpha_{\beta\gamma}$ in terms of the derivative of the metric tensor. Furthermore, using $\overset{\circ}{\nabla}_\mu g_{\rho\sigma} = 0$ in Eq. (3.1) gives the symmetry property of the contortion tensor

$$K_{\sigma\mu\rho} = -K_{\rho\mu\sigma}, \tag{3.4}$$

and thus the contortion tensor is antisymmetric with respect to its first and third indices. Next, using Eq. (3.2) in Eq. (3.3), we get

$$T^\alpha_{\beta\gamma} = K^\alpha_{\beta\gamma} - K^\alpha_{\gamma\beta}. \tag{3.5}$$

Equation (3.5) [using the condition (3.4)] can be inverted and contortion can be expressed in terms of torsion as follows:

$$K^\alpha{}_{\beta\gamma} = \frac{1}{2}(T^\alpha{}_{\beta\gamma} + T_\beta{}^\alpha{}_\gamma + T_\gamma{}^\alpha{}_\beta). \tag{3.6}$$

In general relativity the connection is torsion free, but on account of gauging the Poincaré group it becomes endowed with torsion and the spacetime manifold becomes Riemann-Cartan [50,52]. So if we consider a Lagrangian proportional to the Ricci scalar, which can be written as [55]

$$R = \overset{\circ}{R} + [2\nabla_\rho K^{\rho\sigma}_{\cdot\cdot\sigma} + K^{\rho\alpha\nu} K_{\alpha\rho\nu} - K^\rho_{\cdot\rho\alpha} K^{\alpha\sigma}_{\cdot\cdot\sigma}], \tag{3.7}$$

then R gets a contribution from torsion as well. The resulting theory is called Einstein-Cartan-Sciama-Kibble theory [48]. This is a theory which approximately resembles Einstein's general relativity but also predicts additional effects which arise due to torsion.

IV. MINIMAL COUPLING AND TORSION IN POINCARÉ GAUGE THEORY

The effects of torsion are expected to be very small in the weak-field limit. However, these might play a significant role in the early Universe [56,57]. Introducing electromagnetism and other gauge interactions in this framework turns out to be difficult because they break gauge invariance. As a remedy, a minimal coupling procedure has been prescribed in Ref. [32]. This model, however, is ruled out by solar observations [43]. We will see that, by coupling torsion with non-Abelian gauge fields, this problem can be evaded. We also find it fascinating that the model of Ref. [32] leads to precisely the same form of interaction [i.e., Eq. (2.1)] that was proposed by Ratra.

A. Torsion and electromagnetism

In the presence of torsion, the electromagnetic gauge-covariant derivative can be expressed as

$$\begin{aligned}\nabla_\mu A_\nu &= \partial_\mu A_\nu - \Gamma^\alpha{}_{\mu\nu} A_\alpha \\ &= \partial_\mu A_\nu - \overset{\circ}{\Gamma}{}^\alpha{}_{\mu\nu} A_\alpha - K^\alpha{}_{\mu\nu} A_\alpha = \overset{\circ}{\nabla}_\mu A_\nu - K^\alpha{}_{\mu\nu} A_\alpha.\end{aligned} \tag{4.1}$$

Furthermore, using this, the electromagnetic field tensor can be written as

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu - T^\alpha{}_{\mu\nu} A_\alpha. \tag{4.2}$$

Due to an extra term proportional to the torsion, electromagnetic gauge invariance is not preserved. It has been suggested [32] that if we choose a specific form for the torsion, and slightly modify the gauge transformation conditions, we can restore gauge invariance and still have a restricted version of torsion. We impose the following form of torsion:

$$T^\alpha{}_{\beta\gamma} = \delta^\alpha_\gamma \partial_\beta \phi - \delta^\alpha_\beta \partial_\gamma \phi, \tag{4.3}$$

where ϕ is a scalar field. Using Eq. (4.3) in Eq. (3.6), the contortion has the following form:

$$K^\alpha{}_{\beta\gamma} = g_{\beta\gamma} \partial^\alpha \phi - \delta^\alpha_\beta \partial_\gamma \phi.$$

Using this expression in Eq. (3.7), the Ricci scalar in terms of ϕ is found to be

$$R = \overset{\circ}{R} - 6\partial_\mu \phi \partial^\mu \phi + \frac{2}{\sqrt{-g}} \partial_\rho (\sqrt{-g} K^{\rho\sigma}_{\cdot\cdot\sigma}), \tag{4.4}$$

where $\overset{\circ}{R}$ is the standard Ricci scalar computed using the Christoffel connection $\overset{\circ}{\Gamma}{}^\alpha{}_{\beta\gamma}$. The gauge transformation gets modified to

$$A_\mu \to A_\mu + e^\phi \partial_\mu \varepsilon, \tag{4.5}$$

where ε is the transformation parameter. We see that we obtain an extra contribution equal to e^ϕ which vanishes in the absence of torsion. In this case, it can be easily checked that the field tensor (4.2) and hence the Lagrangian remain invariant.

The modified form of the gauge transformation can now be extended to charged matter fields. Let ψ denote a complex scalar field which transforms under a U(1) gauge transformation as $\psi \to \psi' = \exp(ie\varepsilon)\psi$. The minimal coupling prescription [32] leads to the covariant derivative $D_\mu \psi = \partial_\mu \psi - ie \exp(-\phi) A_\mu \psi$. Hence, we find that the effective electromagnetic coupling in this case is $e/\exp(\phi)$. Using the modified form of torsion in the gravitational Lagrangian, we obtain

$$\begin{aligned}S = \int d^4x \sqrt{-g} \Big[&-\frac{M_{\rm pl}^2}{16\pi} (\overset{\circ}{R} - 6\partial^\rho \phi \partial_\rho \phi) \\ &- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D_\mu \psi)^* D^\mu \psi \Big],\end{aligned} \tag{4.6}$$

where the effects of torsion have been explicitly displayed in terms of the field ϕ, and $M_{\rm pl}$ is the Planck mass. Here we have performed an integration by parts and dropped a total divergence. In order to relate this to the form given in Eq. (2.1), we perform the transformation

$$A^\mu = V^\mu e^\phi. \tag{4.7}$$

In terms of the field V_μ, the gauge transformation of Eq. (4.5) becomes $V_\mu \to V_\mu + \partial_\mu \varepsilon$ and the field tensor becomes

$$F_{\mu\nu}(A) = e^\phi F_{\mu\nu}(V). \tag{4.8}$$

The covariant derivative $D_\mu \psi = \partial_\mu \psi - ieA_\mu \psi$ now takes the standard form and the action can be written as

$$\begin{aligned}S = \int d^4x \sqrt{-g} \Big[&-\frac{M_{\rm pl}^2}{16\pi} (\overset{\circ}{R} - 6\partial^\rho \phi \partial_\rho \phi) \\ &- \frac{1}{4} e^{2\phi} F^{\mu\nu} F_{\mu\nu} + (D_\mu \psi)^* D^\mu \psi \Big].\end{aligned} \tag{4.9}$$

Now, we can see the similarity with the Lagrangian in the Ratra model [14] during inflation. It is clear that the coupling term f^2—which had to be put in by hand in the Ratra model—automatically arises. We point out that this term could be inserted into the coupling to matter fields or directly as a coupling with the gauge kinetic term in the Ratra model, also exactly in analogy with Eqs. (4.6) and (4.9).

However, the Hojman *et al.* minimal prescription model [32] [Eq. (4.9)] is in conflict with solar data [43]. As we shall see in the next two subsections, this problem is evaded since the minimal coupling procedure extended to non-Abelian gauge fields naturally generates effective potential terms for the field ϕ. Alternatively, we may go beyond the minimal coupling prescription [32] by adding kinetic and potential terms in the field ϕ while preserving Poincaré gauge invariance.

Before we end this subsection we briefly discuss an alternative scenario that has been proposed for handling internal gauge symmetries in the case of Poincaré theory. It has been argued in the literature that gauge fields associated with internal symmetries, such as the photon field, should not be coupled to torsion [33,34,58]. This is another way to avoid the problem of the violation of gauge invariance associated with internal groups. However, a massive spin-1 field must necessarily couple to torsion. This presents a paradox since a massive spin-1 field can also be generated from an internal gauge symmetry by the Higgs mechanism. For example, let us consider a U(1) gauge field coupled to a charged scalar particle. Let us use the standard version of the gauge-covariant derivative, $\nabla_\mu A_\nu = \partial_\mu A_\nu$, i.e., we do not introduce the contortion tensor in this definition following the prescription given in Ref. [34]. In contrast, if we were to consider a massive spin-1 field we would need to introduce the contortion term [58].

Now let us consider the coupling of the U(1) gauge field with a charged scalar. The Lagrangian is

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} (D_\mu \phi)^* D_\nu \phi - V, \tag{4.10}$$

where $D_\mu \phi = (\partial_\mu - ieA_\mu)\phi$ is the U(1) gauge-covariant derivative. We assume the Higgs mechanism, i.e., the potential is such that the vacuum value of ϕ is not zero, $\langle \phi \rangle = v$. In this case we find that the gauge field A_μ becomes massive. In the unitary gauge this field behaves identically to a massive vector field. But the problem is that it does not couple to the torsion tensor. Hence, it does not behave as the usual massive spin-1 field. This is clearly inconsistent since a massive vector field must necessarily couple to torsion. In this case, we will have to further generalize our prescription in order to forbid some class of massive spin-1 fields from coupling to torsion. Hence, we argue that it is better to explore the framework presented in Ref. [32] since it does not lead to such inconsistencies.

B. Torsion and non-Abelian fields

In this section we generalize the principle of minimal coupling [32] to make torsion compatible with non-Abelian fields as well. Consider a matter field ψ which under a non-Abelian group G transforms as

$$\psi'(x) = U(x)\psi(x), \tag{4.11}$$

where $U \in G$ is a non-Abelian transformation given by

$$U(x) = \exp[i\alpha^i(x) T^i], \tag{4.12}$$

the α^i's are transformation parameters and the T^i's are generators of the group G. In analogy with Eq. (4.5), we propose the following transformation for the gauge fields W^i:

$$W_\mu^{i\prime} T^i = U\left(W_\mu^i T^i + e^\phi \frac{i}{g} \partial_\mu \right) U^\dagger, \tag{4.13}$$

with the gauge derivative given by

$$D_\mu \psi = (\partial_\mu - ige^{-\phi} W_\mu^i T^i)\psi. \tag{4.14}$$

The covariant derivative transforms as

$$(D_\mu \psi)' = U(x) D_\mu \psi. \tag{4.15}$$

Furthermore, we obtain the field-strength tensor by computing $[D_\mu, D_\nu]$. This leads to

$$[D_\mu, D_\nu]\psi = -ige^{-\phi} F_{\mu\nu}^i T^i \psi, \tag{4.16}$$

where

$$F_{\mu\nu}^i T^i = \partial_\mu W_\nu^i T^i - \partial_\nu W_\mu^i T^i - ige^{-\phi}[W_\mu^i T^i, W_\nu^j T^j] - W_\nu^i T^i \partial_\mu \phi + W_\mu^i T^i \partial_\nu \phi. \tag{4.17}$$

One can easily check that this is just the expression obtained by replacing the derivatives by gravitational gauge-covariant derivatives [see Eq. (4.1)], i.e.,

$$F_{\mu\nu}^i T^i = \nabla_\mu W_\nu^i T^i - \nabla_\nu W_\mu^i T^i - ige^{-\phi}[W_\mu^i T^i, W_\nu^j T^j]. \tag{4.18}$$

We can now make the transformation of the non-Abelian vector potential W_μ^i analogous to Eq. (4.7). This leads to the standard form of the field-strength tensor up to an overall factor of e^ϕ. Hence, the kinetic energy term of the non-Abelian fields becomes $-(1/4)e^{2\phi} F_{\mu\nu}^i F^{i\mu\nu}$ as in the case of Abelian gauge theory.

The coupling of ϕ with non-Abelian gauge fields is interesting since it generates an effective potential for ϕ. Consider a SU(3) gauge field analogous to QCD. After dynamical symmetry breaking, the operator $F_{\mu\nu}F^{\mu\nu}$ acquires an expectation value, i.e.,

$$\langle F_{\mu\nu} F^{\mu\nu} \rangle = \Lambda^4, \tag{4.19}$$

where Λ is a parameter. At leading order we may therefore replace the term in the Lagrangian with

$$e^{2\phi} F_{\mu\nu} F^{\mu\nu} \to e^{2\phi} \langle F_{\mu\nu} F^{\mu\nu} \rangle, \tag{4.20}$$

which leads to an effective potential term for ϕ. We will also get an additional contribution from the topological term $\varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$ which will also pick up an overall factor of $\exp(2\phi)$. We point out that other potential terms are also generated, the details of which depend on the model being considered. We shall discuss an explicit model below.

The potential terms are expected to lead to a background value ϕ_0 of the field ϕ. As the Universe evolves, we assume that ϕ also undergoes a slow cosmological evolution and takes a value ϕ_0 at the current time. The evolution should be sufficiently slow so as not to be in conflict with constraints on the time dependence of fundamental parameters. We next expand ϕ about its background value such that

$$\phi = \phi_0 + \hat{\phi}. \tag{4.21}$$

Furthermore, we scale ϕ in order to convert its kinetic energy term into the canonical form. The background factor e^{ϕ_0} gets absorbed into the redefinition of the parameters. With this expansion we obtain a mass term for $\hat{\phi}$ as well as higher-order terms. This field acquires an effective mass given by

$$m_\phi \sim \frac{\Lambda^2}{\beta M_{\rm pl}}. \tag{4.22}$$

Hence, we generate a mass term as well as higher-order potential terms for the field ϕ. We discuss this in more detail below.

C. Evading the solar constraints

As discussed in the literature [43], the minimal coupling model discussed above is in conflict with constraints from the Solar System. It turns out that the torsion or the scalar field ϕ generated by the Sun is sufficiently large that it leads to observable differences between the gravitational accelerations of particles with different electromagnetic energy content. The nonobservation of this difference rules out the minimal coupling model [32] discussed above. However, the constraints can be evaded if we add a suitable potential $V(\phi)$ to the action such that the scalar field acquires mass. As we have shown in the previous section, we may not need to explicitly add potential terms since an effective potential also gets generated by non-Abelian gauge fields. Once the field acquires mass, its equation of motion can be expressed as

$$\nabla^2\phi - m_\phi^2\phi = \frac{1}{3}(\mathbf{B}^2 - \mathbf{E}^2). \tag{4.23}$$

This is a generalization of Eq. 29 of Ref. [43]. The electromagnetic energy content of the Sun acts as a source of the field ϕ. We see that if the mass m_ϕ is sufficiently large then the field ϕ will decay exponentially and will be negligible at Earth. Assuming that β in Eq. (4.22) is of order unity and Λ is of order 1 GeV (corresponding to the QCD scale), we obtain a value of m_ϕ of order 10^{-10} eV. This is large enough to completely suppress the signal arising due to the Sun.

We point out that besides the potential term generated through QCD [Eq. (4.20)], quantum corrections due to electroweak, gravity, and other beyond the Standard Model fields would also generate other terms in the effective potential for the ϕ field. Hence, it is natural to include potential terms for this field.

V. GENERALIZED GRAVITATIONAL ACTION

The minimal coupling model implied by the Poincaré symmetry has the necessary ingredients to generate magnetogenesis. It naturally produces the coupling of a scalar field ϕ with the electromagnetic field similar to that assumed in Ref. [14]. As we have discussed in Sec. IV B, a potential for the field ϕ also gets generated naturally. Here we shall not necessarily assume that ϕ is also the inflaton field. It is primarily responsible for magnetogenesis. In the next section, we shall study the generation of magnetic fields within the minimal model. As we shall see, the minimal model (4.9) can lead to an enhancement of the magnetic fields. In this section, we study generalized models which may lead to a modification of the kinetic energy term of the field ϕ. This may be useful in avoiding the backreaction problems.

Before discussing the generalized models, we point out that it is possible to add kinetic and potential terms to the field ϕ while preserving Poincaré and electromagnetic gauge invariance, which are the guiding principles in constructing this action. In particular, we can add terms to the gravitational action including the field ϕ, such as

$$L_T = \tilde{\beta} g^{\gamma\sigma} T^\alpha_{\beta\gamma} T^\beta_{\alpha\sigma}. \tag{5.1}$$

This directly leads to a kinetic term for ϕ. In contrast, we cannot generate a potential term for ϕ by adding terms involving the torsion tensor. Such terms have to be added directly in terms of ϕ. However, as we have seen, an effective potential is generated by non-Abelian QCD-like fields. After adding such terms, the final action can be expressed as

$$\begin{aligned} S = \int d^4x\sqrt{-g}\bigg[&-\frac{M_{\rm pl}^2}{16\pi}(\overset{\circ}{R} - 6\beta^2\partial^\rho\phi\partial_\rho\phi) \\ &-\frac{1}{4}e^{2\phi}F^{\mu\nu}F_{\mu\nu} - V(\phi)\bigg], \end{aligned} \tag{5.2}$$

where the added kinetic term has been accommodated by introducing the parameter β.

Following Campanelli [27], we find that this action can generate a magnetic field of the required strength. Depending on the model, i.e., the choice of potential, it may be necessary to choose β to be very small. This will require fine-tuning since the additional kinetic energy term has to be chosen very precisely in order to obtain a small value of β. This fine-tuning may be evaded if we further generalize the gravitational action such that it becomes

$$\begin{aligned} S_g = \int d^4x\sqrt{-g}\bigg[&-e^{2\phi}\frac{M_{\rm pl}^2}{16\pi}R \\ &-\frac{1}{6}e^{2\phi}M_T^2 g^{\mu\nu}T^\alpha{}_{\mu\beta}T^\beta{}_{\alpha\nu} - V(\phi)\bigg]. \end{aligned} \tag{5.3}$$

Here the full form of R is given in Eq. (4.4). We next make a conformal transformation such that

$$\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}. \tag{5.4}$$

In terms of the new metric $\tilde{g}_{\mu\nu}$, the gravitational action becomes

$$S_g = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_{\rm pl}^2}{16\pi}\mathring{R} + \frac{1}{2}M_T^2\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \tag{5.5}$$

Here we do not need to fine-tune the parameter M_T. Now we may choose the potential such that ϕ may act as the inflaton field. The matter action is chosen by the principle of minimal coupling, as prescribed in Ref. [32]. The full action has precisely the form which leads to both inflation and magnetogenesis, following the analysis presented in Refs. [14,27].

Due to the conformal transformation (5.4) the matter action also undergoes some change. The gauge kinetic energy terms remain unaffected. However, terms involving scalar and spinor fields may change. Lets us consider the effect on the Higgs field H. With the transformation (5.4), the Higgs action becomes

$$S_H = \int d^4x\sqrt{-\tilde{g}}[e^{-2\phi}\tilde{g}^{\mu\nu}D_\mu H^\dagger D_\nu H - m_H^2 e^{-4\phi}H^\dagger H - \lambda e^{-4\phi}(H^\dagger H)^2], \tag{5.6}$$

where D_μ is the gauge-covariant derivative. Similar terms are generated for all scalar fields which have nonzero vacuum values. Furthermore, the fermion field condensates are expected to generate additional terms in the effective potential for ϕ. In order to reduce the Higgs kinetic energy term to its canonical form, we may transform the Higgs field such that

$$\tilde{H} = e^{-\phi}H. \tag{5.7}$$

This transformation, however, leads to complicated derivative terms for the ϕ field. In any case, we observe the appearance of additional potential terms for ϕ from the scalar field action besides the terms discussed in Sec. IV B.

Working with the field H, i.e., without transforming to $\tilde{H}$, we may express the potential terms as

$$V(\phi) = ae^{-4\phi} + be^{2\phi} + \cdots \tag{5.8}$$

The factors a and b represent the contributions due to the vacuum expectation value of the Higgs field and the QCD condensates. These are not really constant since these will also change as ϕ evolves with time. In order to determine the vacuum expectation value of ϕ we can treat a and b as constants. Keeping only these two terms, the minimum of the potential is found to be

$$\phi_0 = \frac{1}{6}\ln(2a/b). \tag{5.9}$$

By expanding around the minimum and rescaling ϕ such that $\tilde{\phi} = M_T\phi$, we obtain a mass term for $\tilde{\phi}$. The overall terms, i.e., $\exp(-4\phi_0)$ and $\exp(2\phi_0)$, should be absorbed into the Higgs vacuum expectation value and the QCD condensates, respectively. The ϕ mass is found to be of order

$$\frac{1}{\sqrt{M_T}}\max\{\sqrt{a}, \sqrt{b}\}.$$

We expect a to be at least as large as the electroweak scale and $M_T < M_{\rm pl}$. This generates a mass larger than 10^{-6} eV which is sufficient to evade the solar constraints.

For the generation of primordial magnetic fields, however, the situation is more complicated. This is because now we need to study the time evolution of the Higgs as well as the QCD field. In the case of QCD this will require a rather complicated quantum analysis of non-Abelian fields. We do not pursue this in the present paper and simply assume a potential for ϕ that can lead to primordial magnetogenesis. As discussed earlier, quantum corrections would generate additional terms in the effective potential for ϕ which need to be included for a complete analysis.

The introduction of the factor $e^{2\phi}$ in Eq. (5.3) is partially justified by considering an $f(R)$ gravity model such as the Starobinsky model [59]. In that case the gravitational action becomes very complicated due to the presence of the R^2 term which involves four derivative terms of the field ϕ. However, if we generalize it such that the action reads

$$S_{\rm Staro} = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm pl}^2}{16\pi}\left(e^{2\phi}R - \frac{R^2}{6M^2}\right) + e^{2\phi}M_T^2 g^{\mu\nu}T^\alpha_{\mu\beta}T^\beta_{\alpha\nu}\right], \tag{5.10}$$

then after the conformal transformation the action reduces to the standard Starobinsky action along with an additional kinetic energy term for the field ϕ given in Eq. (5.2). Only with the introduction of this extra factor $e^{2\phi}$ do we get a simple action in the Einstein frame.

An alternative approach is to demand global conformal invariance. In this case the mass scale $M_{\rm pl}$ gets replaced by $a\Phi$, where Φ is a scalar field and a is a parameter; see, for example, Ref. [60]. We do not introduce the extra kinetic energy term proportional to M_T. The resulting gravitational action may be expressed as

$$S_g = \int d^4x\sqrt{-g}\left[\frac{(a\Phi)^2}{16\pi}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi,\phi)\right]. \tag{5.11}$$

In this case we do not need to introduce the extra factor $e^{2\phi}$ in the gravitational action. This action is invariant under the global conformal transformation $g_{\mu\nu} \to g_{\mu\nu}/\Lambda^2$, $\Phi \to \Lambda\Phi$. We also impose this symmetry on the matter action. We next make the transformation $\tilde{g}_{\mu\nu} = e^{2\phi}g_{\mu\nu}$, $\tilde{\Phi} = e^{-\phi}\Phi$. The action now becomes

$$S_g = \int d^4x\sqrt{-\tilde{g}}\left[\frac{(a\tilde{\Phi})^2}{16\pi}\overset{\circ}{R} + \mathcal{L}_K - V(\tilde{\Phi},\phi)\right], \tag{5.12}$$

where

$$\mathcal{L}_K = \frac{1}{2}\tilde{g}^{\mu\nu}(\partial_\mu\tilde{\Phi}\partial_\nu\tilde{\Phi} + \tilde{\Phi}^2\partial_\mu\phi\partial_\nu\phi - 2\tilde{\Phi}\partial_\mu\tilde{\Phi}\partial_\nu\phi). \tag{5.13}$$

The conformal symmetry may be broken softly by the dynamical mechanism analogous to the one described in Refs. [61–65]. Here one assumes the existence of a dark strongly coupled sector. The condensate formation in this sector leads to the dynamical breakdown of conformal invariance, which also triggers the spontaneous electroweak symmetry breaking. Here we assume some strongly coupled sector with a very high mass scale such that these particles decay at some early time during the evolution of the Universe. The condensate formation in this sector also leads to a vacuum expectation value of the field $\tilde{\Phi}$. We assume that this field has undergone negligible evolution since the beginning of inflation, and hence we can simply set it equal to its vacuum expectation value. Alternatively, we need to go to the Einstein frame. In the present case the additional terms generated by going from the Jordan to Einstein frame are assumed to be negligible at leading order due to our assumption that $\tilde{\Phi}$ evolves negligibly during inflation. We therefore set $a\tilde{\Phi}$ equal to its vacuum expectation value which is assumed to be equal to $M_{\rm pl}$. Ignoring the derivatives of $\tilde{\Phi}$, we generate a kinetic energy term for ϕ whose overall normalization is equal to $\tilde{\Phi}^2$. We assume that this value is sufficiently small in comparison to $M_{\rm pl}$ so that it does not lead to backreaction during inflation. This requires us to set the parameter a to be of order 10^3. Hence, with an appropriate choice of the parameter a this leads to exactly the action given in Eq. (5.5) with the scale M_T being generated by $\tilde{\Phi}$.

Before ending this section, we propose a generalization of the Starobinsky model by demanding conformal symmetry. The resulting action can be written as

$$S_C = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi}(a^2\Phi^2R - \xi^2R^2) + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi,\phi)\right]. \tag{5.14}$$

We again make the transformation $\tilde{g}_{\mu\nu} = e^{2\phi}g_{\mu\nu}$, $\tilde{\Phi} = e^{-\phi}\Phi$. The resulting action reads

$$S_C = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{16\pi}(a^2\tilde{\Phi}^2\overset{\circ}{R} - \xi^2\overset{\circ}{R}{}^2) + \mathcal{L}_K - V(\tilde{\Phi},\phi)\right]. \tag{5.15}$$

As discussed earlier, the field $\tilde{\Phi}$ acquires a vacuum expectation value such that $a\tilde{\Phi} = M_p$. Assuming that $\tilde{\Phi}$ does not evolve significantly with time during inflation, we can replace it with its vacuum expectation value. Then the first two terms on the right-hand side yield the standard Starobinsky model. The remaining terms involve the kinetic and potential terms for the fields ϕ and $\tilde{\Phi}$. Along with the coupling with the electromagnetic field, these can lead to magnetogenesis.

To summarize this section, we have shown that the model for magnetogenesis proposed by Ratra [14] naturally appears in the Poincaré-invariant theory with the minimal prescription principle proposed in Ref. [32]. The minimal model, however, requires fine-tuning of the kinetic energy term and does not contain a potential term for the scalar field ϕ, which generates torsion. We have argued that potential terms for this field appear naturally and it is easy to generalize the model such that it does not suffer from fine-tuning. This is best accomplished by imposing conformal invariance. Hence, the model has all of the ingredients required for primordial magnetogenesis. We have also described a conformal generalization of the Starobinsky model that will lead to both inflation and magnetogenesis.

VI. MAGNETOGENESIS

The basic formalism for magnetogenesis in the Ratra model was developed in several papers [25,27]. Our basic point is that Poincaré gauge theory naturally leads to models similar to the Ratra model. In particular, the minimal coupling model leads to a specific form of the coupling of a scalar field ϕ to the electromagnetic fields as well as the kinetic term for ϕ. As we have discussed in the previous section, both the kinetic and potential terms in ϕ may be substantially modified in comparison to what we obtain using the minimal coupling procedure. Hence, depending on the model, different scenarios described in Ref. [27] could be realized and we can generate a magnetic field of the required strength if we go beyond the minimal coupling model. In this section we discuss how successful the minimal coupling model is at generating magnetic fields.

The minimal coupling model, proposed in Ref. [32], is given in Eq. (4.9). Here we have only displayed the coupling of ϕ with the electromagnetic field. Similar terms should also be included for all gauge fields, both Abelian and non-Abelian. As described above, an effective potential term for the scalar field ϕ of the form

$$V = M_1^4 e^{2\phi} \tag{6.1}$$

naturally appears due to the vacuum condensates formed by QCD-like non-Abelian fields. Here we assume the existence of some QCD-like fields with a very high mass scale M_1. The equation of motion for ϕ, neglecting electromagnetic effects, can be expressed as

$$12M_{\rm pl}^2\beta^2(\ddot{\phi} + 3H\dot{\phi}) + \frac{\partial V}{\partial \phi} = 0, \tag{6.2}$$

with $\beta = 1$. Here we assume that inflation is caused by some field other than ϕ. Let us assume that during inflation the time-dependent part of this field is small. In that case, we can approximate $\exp(2\phi) \approx 1 + 2\phi$. This leads to a linear potential $V(\phi) \approx b\phi$ for ϕ and admits a solution of the form

$$e^{\phi} = C\left(\frac{a}{a_i}\right)^{\alpha} \tag{6.3}$$

with constant $\alpha \ll 1$. Ignoring the constant part of ϕ, we obtain $\phi \approx \alpha Ht$. The solution starts to break down as ϕ approaches unity. This corresponds to the small-p (p is same as α in our notation) limit of the models discussed in Ref. [27]. In this limit the model leads to magnetic fields of order 10^{-32} G, which is rather small.

We next discuss the more general case in which the field ϕ is not necessarily small while working within the framework of the minimal coupling model. In this case we use the full form of the potential given in Eq. (6.1). Assuming that $\ddot{\phi}$ is negligible, the solution for ϕ becomes

$$e^{2\phi} = \frac{1}{2m(t + t_0)}, \tag{6.4}$$

where $m = M_1^4/(18M_{\rm pl}^2 H)$ and t_0 is an integration constant. We set $t_0 = N_0/H$ and find that $\ddot{\phi}$ is negligible if

$$Ht + N_0 \gg 1, \tag{6.5}$$

which holds for a wide range of choices of N_0. We point out that this requires N_0 to be sufficiently larger than unity. Furthermore, this condition holds independent of the value of β. With this condition we also find that the kinetic energy term for ϕ does not cause backreaction for the inflationary potential. The effective value of α in this case is found to be

$$\alpha = -\frac{1}{2(Ht + N_0)}, \tag{6.6}$$

which varies slowly with time but is necessarily small throughout inflation. Hence, we find that with this choice of potential we are forced to have small values of α which effectively also imply a small value of the time-dependent part of ϕ. This implies that the minimal coupling model does not lead to sufficiently large magnetic fields which can provide seeds for the galactic dynamo. This can be modified only if we allow a generalized potential which is permissible in our framework but goes beyond the minimal coupling model.

So far we have assumed that ϕ is not the inflaton field. However, since it varies slowly, the effective potential remains approximately constant during most of the evolution. Hence, it is also possible to consider ϕ as the inflaton field, as long as we choose N_0 to be sufficiently large. However, it is not clear how to exit inflation and enter the reheating phase in this framework. It may be possible to enhance the model in order to accomplish this. For example, we may also add the term $F^{\mu\nu}\tilde{F}_{\mu\nu}$ for the non-Abelian fields which also acquires a vacuum expectation value. This term has an effective coupling to an axion-like field χ and would also pick up a factor of $e^{2\phi}$. Let us assume that this condensate dominates the evolution during inflation. Furthermore, the axion field χ remains constant during inflation but undergoes evolution towards the end, which effectively ends inflation and leads to reheating.

A. Beyond the minimal model

We have already seen that within the framework of the minimal model we are unable to generate a magnetic field of the required strength. As discussed earlier, the required kinetic energy term in ϕ can be generated naturally within the minimal model, as demonstrated, for example, in Eqs. (5.11) and (5.12). However, we are unable to generate the required potential term. Here we go beyond the minimal model by adding the following potential term:

$$V(\phi) = 6\beta^2 M_p^2 m^2 \phi^2, \tag{6.7}$$

where m is a parameter with dimensions of mass. The kinetic energy term in ϕ is given in Eq. (5.2), or equivalently in Eq. (5.13) with $\langle\tilde{\Phi}\rangle = \sqrt{3/4\pi}\beta M_{pl}$. In this case ϕ also acts as the inflaton.

Imposing the slow-roll condition $\ddot{\phi} \ll 3H\dot{\phi}$ in the equation of motion (6.2), we obtain the solution

$$\phi = \phi_0 e^{-t/\tau}, \tag{6.8}$$

where ϕ_0 and τ are constants, such that

$$\tau = \frac{3H}{m^2}. \tag{6.9}$$

The slow-roll condition implies

$$\tau \gg \frac{1}{H}, \tag{6.10}$$

which also leads to

$$\frac{m^2}{H^2} \ll 1. \tag{6.11}$$

From Eq. (6.3), we obtain

$$\alpha = -\frac{\phi_0}{\tau H} e^{-t/\tau}. \tag{6.12}$$

Now, let us estimate the value of ϕ_0. Einstein's equations of motion give us

$$3H^2 = \frac{3}{2}\beta^2\dot{\phi}^2 + 3\beta^2 m^2 \phi^2. \tag{6.13}$$

Here, in order for the kinetic term to be negligible, we need

$$\tau \gg \frac{1}{m}. \tag{6.14}$$

Ignoring the kinetic energy term, we obtain $\phi_0 \approx H/(\beta m)$.

Let us now assume that $\alpha \approx -2$ and is approximately constant over much of inflation. The value $\alpha = -2$ is required in order to generate a scale-invariant magnetic field within the framework developed in Ref. [27]. This can be accomplished by requiring that $t \ll \tau$ during inflation. This is equivalent to requiring that $H\tau \gg N$, where N is the number of e-folds during inflation. For example, taking $H\tau \approx 500$, assuming that $N \approx 60$, will ensure that the necessary conditions are met over much of inflation and that we produce a nearly scale-invariant spectrum for a wide range of values of k of the required strength [27]. The parameter values for this case would be $m \approx H\sqrt{3/500}$ and $\beta \approx 1/(-\alpha\tau m) \approx 0.5/\sqrt{1500} \ll 1$. Hence, with this choice of parameters we are able to generate the required magnetic field.

VII. CONCLUSIONS

The problem of generating magnetic fields during inflation can be resolved by breaking the conformal invariance of the electromagnetic Lagrangian [14]. The required coupling between torsion and electromagnetism, however, has to be put in by hand. We have shown that torsion naturally leads to this coupling within the framework of the model developed by Hojman *et al.* [32], which satisfies electromagnetic gauge invariance. The model is based on a minimal version of torsion such that the torsion field can be expressed in terms of a scalar field ϕ, called the tlaplon in the literature [32]. The main problem with this model is that it is ruled out by constraints due to solar data [43]. We have shown that, due to coupling with non-Abelian fields, the minimal model acquires an effective potential term for the scalar field and evades the solar constraints. We have studied the generation of magnetic fields within the framework of the minimal model. We found that these are equivalent to the small-p (or α) limit of the models discussed in Ref. [27]. Hence, the magnetic field generated in this case is relatively small. The minimal model also leads to a rather large contribution from the kinetic energy of ϕ unless p is close to 0.

We have also discussed several generalizations of the minimal model. We have argued that quantum corrections will generate additional potential terms and hence there is no reason to restrict oneself to the minimal model. We can add potential terms to the scalar tlaplon while maintaining Poincaré symmetry. With these terms it is possible to generate larger magnetic fields in comparison to the minimal model. In particular, we have considered a model which displays invariance under the global conformal transformation $g_{\mu\nu} \to g_{\mu\nu}/\Lambda^2$, $\Phi \to \Lambda\Phi$, where Φ is a scalar field. By imposing this symmetry on a generalized Starobinsky model, we found that we can naturally suppress the kinetic energy terms of ϕ. With this suppression it possible to have significant variation of ϕ during inflation, which is required for magnetogenesis. By assuming a simple form of the potential for ϕ, we have explicitly demonstrated the generation of a nearly scale-invariant magnetic field of the required strength. However, we still need a mechanism to generate the required potential which has so far simply been assumed. For this purpose it may be useful to study the conformal model in more detail using the effective potential approach, which includes corrections due to loop contributions.

ACKNOWLEDGMENTS

We thank Bharat Ratra for useful comments on the paper. R. K. is supported by the South African Radio Astronomy Observatory (SA RAO) and the National Research Foundation (Grant No. 75415).

[1] L. M. Widrow, D. Ryu, D. R. G. Schleicher, K. Subramanian, C. G. Tsagas, and R. A. Treumann, Space Sci. Rev. **166**, 37 (2012).
[2] K. Subramanian, Rep. Prog. Phys. **79**, 076901 (2016).
[3] K. E. Kunze, Plasma Phys. Controlled Fusion **55**, 124026 (2013).
[4] A. Neronov and I. Vovk, Science **328**, 73 (2010).
[5] F. Tavecchio, G. Ghisellini, L. Foschini, G. Bonnoli, G. Ghirlanda, and P. Coppi, Mon. Not. R. Astron. Soc. **406**, L70 (2010).
[6] F. Tavecchio, G. Ghisellini, G. Bonnoli, and L. Foschini, Mon. Not. R. Astron. Soc. **414**, 3566 (2011).
[7] C. Caprini and S. Gabici, Phys. Rev. D **91**, 123514 (2015).
[8] T. Kahniashvili, A. G. Tevzadze, S. K. Sethi, K. Pandey, and B. Ratra, Phys. Rev. D **82**, 083005 (2010).
[9] K. Ferrière, Astron. Astrophys. **505**, 1183 (2009).
[10] M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743 (1988).
[11] A. Kandus, K. E. Kunze, and C. G. Tsagas, Phys. Rep. **505**, 1 (2011).
[12] R. Durrer and A. Neronov, Astron. Astrophys. Rev. **21**, 62 (2013).
[13] H. Moffatt, Adv. Appl. Mech. **16**, 119 (1976).
[14] B. Ratra, Astrophys. J. Lett. **391**, L1 (1992).
[15] A. Dolgov, Phys. Rev. D **48**, 2499 (1993).
[16] M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Rev. Lett. **75**, 3796 (1995).
[17] M. Giovannini, Phys. Rev. D **62**, 123505 (2000).
[18] J. Martin and J. Yokoyama, J. Cosmol. Astropart. Phys. 01 (2008) 025.
[19] K. Subramanian, Astron. Nachr. **331**, 110 (2010).
[20] K. Atmjeet, I. Pahwa, T. R. Seshadri, and K. Subramanian, Phys. Rev. D **89**, 063002 (2014).
[21] T. Fujita, R. Namba, Y. Tada, N. Takeda, and H. Tashiro, J. Cosmol. Astropart. Phys. 05 (2015) 054.
[22] D. Nandi and S. Shankaranarayanan, J. Cosmol. Astropart. Phys. 01 (2018) 039.
[23] V. Demozzi, V. Mukhanov, and H. Rubinstein, J. Cosmol. Astropart. Phys. 08 (2009) 025.
[24] T. Fujita and S. Mukohyama, J. Cosmol. Astropart. Phys. 10 (2012) 034.
[25] R. J. Z. Ferreira, R. K. Jain, and M. S. Sloth, J. Cosmol. Astropart. Phys. 10 (2013) 004.
[26] C. G. Tsagas, Symmetry **8**, 122 (2016).
[27] L. Campanelli, Phys. Rev. D **93**, 063501 (2016).
[28] R. K. Jain and M. S. Sloth, J. Cosmol. Astropart. Phys. 02 (2013) 003.
[29] R. J. Z. Ferreira, R. K. Jain, and M. S. Sloth, J. Cosmol. Astropart. Phys. 06 (2014) 053.
[30] T. R. Seshadri and K. Subramanian, Phys. Rev. Lett. **103**, 081303 (2009).
[31] T. W. B. Kibble, J. Math. Phys. (N.Y.) **2**, 212 (1961).
[32] S. Hojman, M. Rosenbaum, M. P. Ryan, and L. C. Shepley, Phys. Rev. D **17**, 3141 (1978).
[33] F. W. Hehl, Gen. Relativ. Gravit. **4**, 333 (1973).
[34] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976).
[35] I. J. Muzinich, J. Math. Phys. (N.Y.) **26**, 1942 (1985).
[36] G. German, Classical Quantum Gravity **2**, 455 (1985).
[37] M. O. Katanaev and I. V. Volovich, Phys. Lett. **156B**, 327 (1985).
[38] C. H. Oh and K. Singh, Classical Quantum Gravity **6**, 1053 (1989).
[39] G. German, A. Macias, and O. Obregon, Classical Quantum Gravity **10**, 1045 (1993).
[40] G.-M. Zhang and Y.-L. Wu, Int. J. Theor. Phys. **35**, 1211 (1996).
[41] C. Kohler, Int. J. Mod. Phys. A **15**, 1235 (2000).
[42] K. Hayashi and T. Shirafuji, Prog. Theor. Phys. **64**, 866 (1980); **65**, 2079(E) (1981).
[43] W.-T. Ni, Phys. Rev. D **19**, 2260 (1979).
[44] D. Gorbunov and V. Rubakov, *Introduction to the Theory of the Early Universe: Cosmological Perturbations and Inflationary Theory* (World Scientific, Singapore, 2011), ISBN: 9789814343787.
[45] N. Birrell and P. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 1984), ISBN: 9780521278584.
[46] F. Mansouri and L. N. Chang, Phys. Rev. D **13**, 3192 (1976).
[47] F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne'eman, Phys. Rep. **258**, 1 (1995).
[48] N. Popławski, Phys. Rev. D **85**, 107502 (2012).
[49] Semi direct product is a special case of direct product of two groups, when one of the groups under consideration happens to be a normal group. In the present case $SO(1,3)$ is the normal subgroup of the Poincaré group.
[50] S. A. Ali, C. Cafaro, S. Capozziello, and C. Corda, Int. J. Theor. Phys. **48**, 3426 (2009).
[51] R. Utiyama, Phys. Rev. **101**, 1597 (1956).
[52] F. W. Hehl, Einstein Stud. **13**, 145 (2017).
[53] M. Blagojević and F. Hehl, *Gauge Theories of Gravitation, A Reader with Commentaries* (World Scientific, Singapore, 2013).
[54] S. Carroll, *Spacetime and Geometry: An Introduction to General Relativity*, Always learning (Pearson Education, London, England, 2013), ISBN: 9781292026633.
[55] I. L. Shapiro, Phys. Rep. **357**, 113 (2002).
[56] N. J. Popławski, Gen. Relativ. Gravit. **44**, 1007 (2012).
[57] F. W. Hehl, P. V. der Heyde, and G. D. Kerlick, Phys. Rev. D **10**, 1066 (1974).
[58] M. Blagojević, SFIN A, contribution to 2nd Summer School in Modern Mathematical Physics (MPHYS2) **1**, 147 (2003).
[59] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[60] P. Jain, P. Karmakar, S. Mitra, S. Panda, and N. K. Singh, Classical Quantum Gravity **28**, 215010 (2011).
[61] T. Hur and P. Ko, Phys. Rev. Lett. **106**, 141802 (2011).
[62] K. A. Meissner and H. Nicolai, Phys. Lett. B **648**, 312 (2007).
[63] K. A. Meissner and H. Nicolai, Phys. Lett. B **660**, 260 (2008).
[64] G. Kashyap, S. Mitra, and P. Jain, Astropart. Phys. **75**, 64 (2016).
[65] P. Jain, G. Kashyap, and S. Mitra, Int. J. Mod. Phys. A **30**, 1550171 (2015).