New formulas for the (−2) moment of the photoabsorption cross section, σ_{-2}

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Two new formulas for the (−2) moment of the photoabsorption cross section, σ_{-2}, have been determined, respectively, from the 1988 photoneutron evaluation of Dietrich and Berman and a mass-dependent symmetry energy coefficient, $a_{\rm sym}(A)$. The data for $A \gtrsim 50$ follow, with a RMS deviation of 6%, the power law $\sigma_{-2} = 2.4A^{5/3}$ μb/MeV, which is in agreement with Migdal's calculation of $\sigma_{-2} = 2.25A^{5/3}$ μb/MeV based on the hydrodynamic model and the σ_{-2} sum rule. The additional inclusion of $a_{\rm sym}(A)$ provides a deeper insight into the nuclear polarization of $A \geqq 10$ nuclei.

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The ratio of the induced dipole moment to an applied constant electric field yields the static nuclear polarizability, α. On using the hydrodynamic model and assuming interpenetrating proton and neutron fluids with a well-defined nuclear surface of radius $R = r_0 A^{1/3}$ fm, Migdal [1,2] obtains

$$\alpha = \frac{e^2 R^2 A}{40 a_{\rm sym}} = 2.25 \times 10^{-3} A^{5/3}\ {\rm fm}^3, \tag{1}$$

where $a_{\rm sym} = 23$ MeV is the symmetry energy coefficient in the Bethe-Weizsäcker semiempirical mass formula [3,4] and $r_0 = 1.2$ fm. This semiclassical treatment considers the nuclear symmetry energy, $a_{\rm sym}(N-Z)^2/A$, to be spread uniformly throughout the nucleus as a symmetry energy density $a_{\rm sym}(\rho_n - \rho_p)^2/\rho$.

Alternatively, α can be calculated from the (−2) moment of the total electric-dipole photoabsorption cross section, σ_{-2},

$$\sigma_{-2} = \int_0^\infty \frac{\sigma_{\rm total}(E_\gamma)}{E_\gamma^2} dE_\gamma, \tag{2}$$

using second-order perturbation theory [5,6]. It follows from the sum rule[1]

$$\alpha = 2e^2 \sum_n \frac{\langle i \| \hat{E}1 \| n\rangle \langle n \| \hat{E}1 \| i\rangle}{E_\gamma} \tag{3}$$

$$= \frac{e^2\hbar^2}{M} \sum_n \frac{f_{\rm in}}{E_\gamma^2} = \frac{\hbar c}{2\pi^2} \int_0^\infty \frac{\sigma_{\rm total}(E_\gamma)}{E_\gamma^2} dE_\gamma \tag{4}$$

$$= \frac{\hbar c}{2\pi^2} \sigma_{-2}, \tag{5}$$

where E_γ is the γ-ray energy corresponding to a transition connecting the ground state $|i\rangle$ and an excited state $|n\rangle$, M the nucleon mass, and $\sigma_{\rm total}(E_\gamma)$ the total photoabsorption cross section. The $\sigma_{\rm total}(E_\gamma)$ cross section generally includes the $(\gamma,n) + (\gamma,p) + (\gamma,np) + (\gamma,2n) + (\gamma,3n) + (\gamma,F)$ channels, which are in competition in the giant dipole resonance (GDR) region [9,10].

On comparing Eqs. (1) and (5), Migdal extracted σ_{-2} as [1]

$$\sigma_{-2} = 2.25 A^{5/3}\ \mu{\rm b/MeV}. \tag{6}$$

This power-law relationship was empirically confirmed by Levinger in 1957 from a fit to the available σ_{-2} data [2],

$$\sigma_{-2} = 3.5\kappa A^{5/3}\ \mu{\rm b/MeV}. \tag{7}$$

Levinger's fit is shown in Fig. 1 and included eleven σ_{-2} data points (squares) with approximate estimations for the high-energy, neutron multiplicity and $\sigma(\gamma,p)$ contributions. The polarizability parameter κ is the ratio of the observed GDR effect to that predicted by the hydrodynamic model [2], as determined by comparing the measured σ_{-2} values and Eq. (7). This comparison yields $\kappa = 1$ for the ground state of nuclei with $A \gtrsim 20$ [2]. Lighter nuclei require larger values of κ to reproduce the data. Using Eqs. (5) and (7), the nuclear polarizability is given by

$$\alpha = 3.5k \times 10^{-3} A^{5/3}\ {\rm fm}^3, \tag{8}$$

which depends on the nuclear size and κ.

In 1988, Dietrich and Berman re-evaluated the photoneutron cross-section data [11]. This evaluation included $(\gamma,n) + (\gamma,pn) + (\gamma,2n) + (\gamma,3n) + (\gamma,F)$ data from studies at Livermore, Giessen, Saclay, and other laboratories which used monochromatic photon beams generated by in-flight annihilation of positrons.[2]

Figure 1 shows the σ_{-2} data (in μb/MeV) from the Dietrich and Berman evaluation (circles) [11], by integrating Eq. (2) between the (γ,n) threshold and an upper limit of $E_{\gamma_{\rm max}} \approx$ 20–50 MeV. These integration limits include the GDR but do not take into consideration $\sigma(\gamma,p)$ contributions and the rise of $\sigma(E_\gamma)$ at around 140 MeVdue to pion exchange currents [12].

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[1]The dimensionless oscillator strength $f_{\rm in}$ for $E1$ transitions, $f_{\rm in} = \frac{2M}{\hbar^2} E_\gamma \langle i \| \hat{E}1 \| n\rangle \langle n \| \hat{E}1 \| i\rangle$, and its relation with the total photoabsorption cross section, $\int_0^\infty \sigma_{\rm total}(E_\gamma) dE_\gamma = \frac{2\pi^2 e^2 \hbar}{Mc} \sum_n f_{\rm in}$, are introduced in Eqs. (3) and (4), respectively [6–8].

[2]Most of the photonuclear data produced during 1960–1988 was taken with monochromatic photon beams [10,11]. One main advantage of this technique over bremsstrahlung photon beams, broadly used prior to 1960, is the direct and simultaneous measurements of the partial photoneutron cross sections which are in competition in the GDR region. These simultaneous measurements are essential to obtain a reliable $\sigma_{\rm total}(E_\gamma)$ [10].

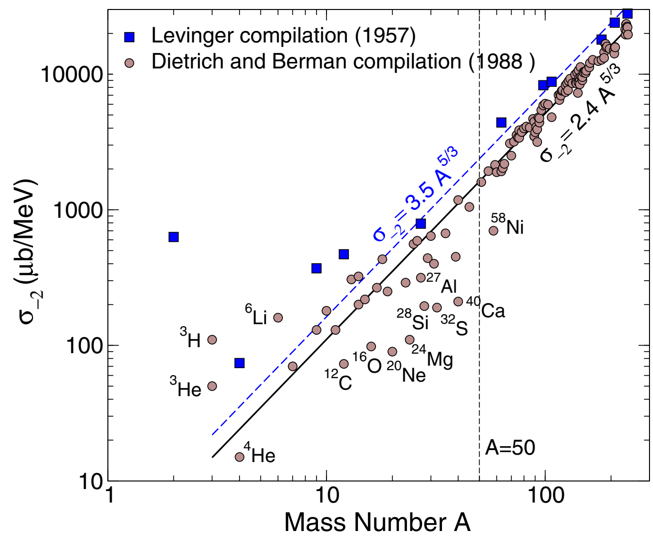

FIG. 1. (Color online) The (−2) moment of the total photoabsorption cross section σ_{-2} vs A on a log-log scale. The experimental values from the 1988 compilation [11] are given by circles. These data follow a power-law relationship $\sigma_{-2} = 2.4A^{5/3}$ μb/MeV. The dashed line represents Levinger's fit to the available data (squares) in 1957, $\sigma_{-2} = 3.5A^{5/3}$ μb/MeV [2]. In both cases, $\kappa = 1$ is assumed.

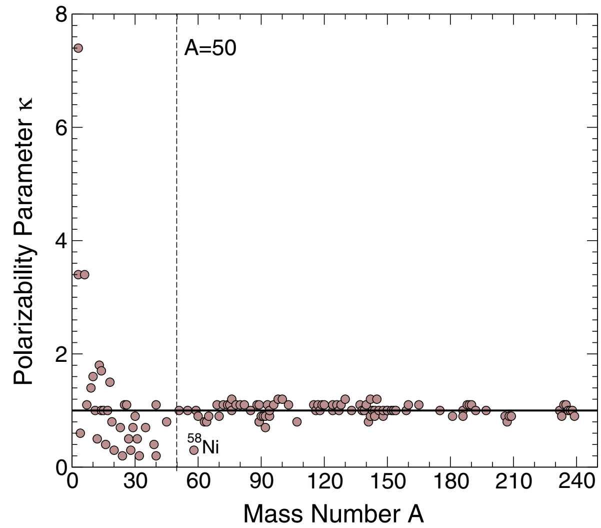

FIG. 2. (Color online) The polarizability parameter κ given by $\frac{\sigma_{-2}}{2.4A^{5/3}}$ using the σ_{-2} data in the Dietrich and Berman compilation (circles) [11]. The horizontal solid line corresponds to $\kappa = 1$. Large deviations from the hydrodynamic model prediction ($\kappa = 1$) are observed for $A \lesssim 50$.

Because of the $1/E_\gamma^2$ factor, σ_{-2} is less sensitive to these high-energy contributions, which account for less than 10% of the total σ_{-2} value [2,12–14]. This plot uses the mean value when several measurements were available for the same isotope and excluded data from natural samples unless one single isotope dominated the isotopic abundance.

These data follow a power law,

$$\sigma_{-2} = 2.4\kappa A^{5/3}\ \mu\text{b/MeV}, \tag{9}$$

with a RMS deviation of 30% for $\kappa = 1$. For $A \geqslant 50$, on excluding the ^{58}Ni data point which has a large $\sigma(\gamma,p)$ contribution [15,16], the agreement is even better, as shown in Fig. 1, with a RMS deviation of 6%. This formula agrees with the one published by Berman and Fultz in their 1975 review paper for $A \gtrsim 60$: $\sigma_{-2} = 2.39(20)A^{5/3}\mu$b/MeV [15]. For $A < 50$, Fig. 1 presents large deviations from $\kappa = 1$ for $A = 4n$, $T_Z = 0$ nuclei ($\kappa < 1$) and loosely-bound light nuclei with $A < 20$ ($\kappa > 1$). To emphasize this point, Fig. 2 shows a similar plot of the polarizability parameter κ vs A by comparing Eq. (9) and the empirical σ_{-2} values [11].

The missing $\sigma(\gamma,p)$ contribution in the Dietrich and Berman evaluation is the reason for the $\kappa < 1$ values observed[3] for many $A < 50$ nuclei and ^{58}Ni. For heavier nuclei, neutron emission is the favorable decay mode due to the strong suppression of proton emission by the Coulomb barrier. Proton emission is, however, the predominant decay mode for $A = 4n$ self-conjugate nuclei with $A \lesssim 50$ [17,18]. For example, $\sigma(\gamma,p) \approx 7 \times \sigma(\gamma,n)$ in ^{40}Ca [19]. This is because of the isospin selection rule $\Delta T = \pm 1$ for $E1$ excitations in a $T_Z = 0$ self-conjugate nucleus.[4] For a nucleus with a ground state of isospin T, there is an isospin splitting of the GDR [21] which corresponds to excited proton $(T+1)$ and neutron (T) resonances, with the $T+1$ resonance generally lying at a higher excitation energy. The isospin selection rule[5] favors the excitation of $T+1$ states [20], which predominantly decay by proton emission[6] [25]. Although the $\sigma(\gamma,p)$ data are scarce, the σ_{-2} sum rule [26] seems to be exhausted once the $\sigma(\gamma,p)$ contributions are included [17,18,27].

The larger GDR effect ($\kappa > 1$) observed in Fig. 2 for light nuclei with $A \lesssim 20$ may be explained from the mass dependence of the symmetry energy coefficient, $a_{\text{sym}}(A)$, of relevance to test $3N$ forces [28] and describe neutron stars and supernova cores [29,30]. As mentioned above, Migdal utilized a constant value of $a_{\text{sym}} = 23$ MeV to determine σ_{-2} in Eq. (6). Nevertheless, the mass dependence of $a_{\text{sym}}(A)$ has long been established in the liquid droplet model [31] and recognized as the fundamental parameter describing the GDR [15]. Its form has since been refined, despite its current

[3]The total photoneutron cross section, $\sigma(\gamma,n)$, for ^{58}Ni is relatively small because of the $\frac{\sigma(\gamma,p)}{\sigma(\gamma,n)}$ ratio is also controlled by the relative level densities in the residual nuclei, i.e., the ratio of the number of open channels, $\frac{N_p}{N_n}$. For ^{58}Ni, $\frac{N_p}{N_n} \approx \frac{\sigma(\gamma,p)}{\sigma(\gamma,n)} \approx 2$ [16].

[4]$\Delta T = \pm 1$ isovector transitions are isospin forbidden as the Wigner coefficient $\begin{pmatrix} T_f & 1 & T_i \\ -T_Z & 0 & T_Z \end{pmatrix}$ vanishes for $T_Z = 0$ [20].

[5]With only small admixtures [22,23], isospin is a good approximation in photonuclear reactions for light nuclei involving photons in the range of the electric dipole absorption [24].

[6]Most neutron emission from excited states with isospin $T+1$ is forbidden, whereas neutron emission from excited states with isospin T is allowed [25]. These selection rules follow from the respective Clebsch-Gordan coefficients in the transition probabilities.

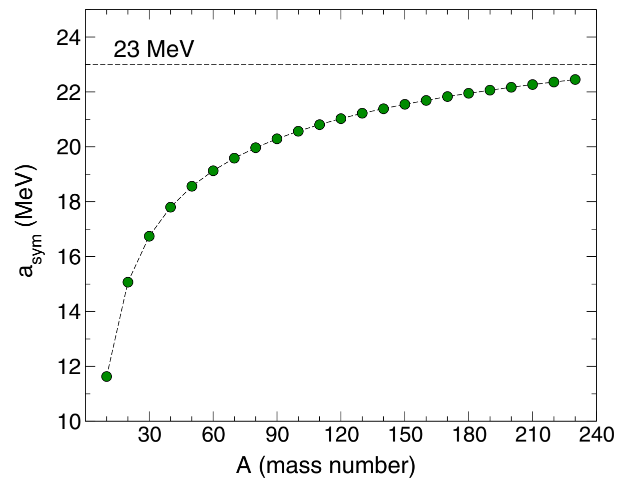

FIG. 3. (Color online) Symmetry energy coefficient, $a_{\rm sym}(A)$, of finite nuclei as a function of mass number A using Eq. (10) [32].

model dependency [32], with the advent of high-precision mass measurements.

From a global fit to the binding energies of isobaric nuclei with $A \geqslant 10$ [32], extracted from the 2012 atomic mass evaluation [33], Tian and co-workers determined $a_{\rm sym}(A)$ as

$$a_{\rm sym}(A) = S_v\left(1 - \frac{S_s}{S_v A^{1/3}}\right), \tag{10}$$

with $S_v \approx 28.32$ MeV being the bulk symmetry energy coefficient and $\frac{S_s}{S_v} \approx 1.27$ the surface-to-volume ratio.[7] Within this approach, the extraction of $a_{\rm sym}(A)$ only depends on the Coulomb energy term in the Bethe-Weizsäcker semiempirical mass formula and shell effects [35], which are both included in Eq. (10) [32]. Figure 3 illustrates the mass dependency of $a_{\rm sym}(A)$ and clearly prevents the use of a constant $a_{\rm sym}$ value.

After introducing this mass dependence in Eqs. (1) and (5), α and σ_{-2} are given by

$$\alpha = \frac{1.8\times 10^{-3}A^2}{A^{1/3}-1.27}\ {\rm fm}^3, \tag{11}$$

$$\sigma_{-2} = \frac{1.8A^2}{A^{1/3}-1.27}\ \mu{\rm b/MeV}. \tag{12}$$

Equation (12) is plotted in Fig. 4 for $A \geqslant 10$ nuclides (solid line). Encouragingly, the increasing upbend observed as A decreases provides an explanation for the large GDR effects observed in light nuclei. However, the validity of the hydrodynamic model remains to be tested for the lightest $A < 10$ nuclei.

More generally, Eq. (12) provides a means to evaluate nuclear polarizability without invoking a polarizability parameter. As shown in Fig. 4, most of the data points either fall below the predicted curve ($A < 70$) or merge with it where neutron emission is favorable ($A \geqslant 70$). These facts indicate that Eq. (12) could exhaust the σ_{-2} sum rule for

[7] Similar coefficients are determined in Ref. [34].

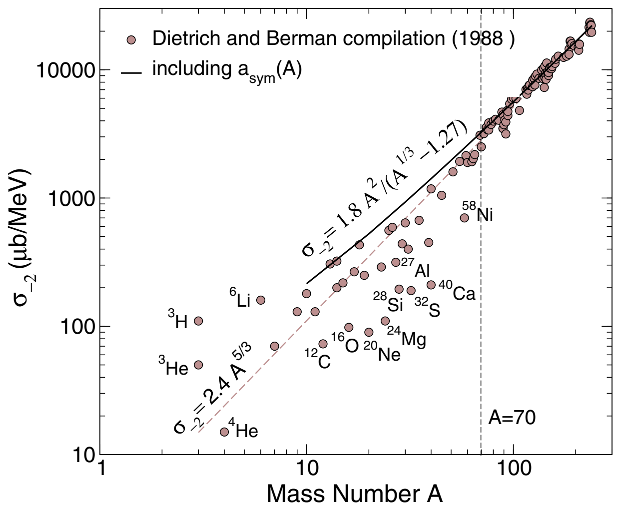

FIG. 4. (Color online) The (−2) moment of the total photoabsorption cross section σ_{-2} vs A on a log-log scale using Eq. (12) (solid line). For comparison purposes, Eq. (9) (dashed line) and the data from the 1988 compilation [11] are also plotted.

both photoneutron and photoproton cross sections and, hence, incorporate the actual GDR effect to the nuclear polarizability. Consequently, the mass-dependent σ_{-2} curve may provide an estimate for the missing $\sigma(\gamma,p)$ contribution. For example, the predicted value of σ_{-2} for ^{40}Ca is in agreement with the experimentally determined $\sigma(\gamma,p)/\sigma(\gamma,n)$ ratio [19]. Additional experimental and theoretical work are needed to test the generality of these findings and evaluate deviations from the hydrodynamic model.

In conclusion, a new empirical formula [Eq. (9)] for the (−2) moment of the photoabsorption cross section, $\sigma_{-2}(A)$, has been determined from the latest photoneutron cross-section evaluation with monoenergetic photons. The σ_{-2} data include most of the photoneutron channels but excludes relevant $\sigma(\gamma,p)$ contributions for $A \lesssim 50$ nuclides. This new empirical formula presents a RMS deviation of 6% for $A \gtrsim 50$ and is in better agreement with Migdal's calculation of σ_{-2} [Eq. (6)] on combining the hydrodynamic model and second-order perturbation theory.

Additionally, σ_{-2} has been inferred [Eq. (12)] using a mass-dependent symmetry energy coefficient, $a_{\rm sym}(A)$, determined by Tian and collaborators for $A \geqslant 10$ nuclei, which includes Coulomb energy and shell corrections. The resulting curve seems to account for the actual GDR effects as it exhausts the σ_{-2} sum rule for most $A \geqslant 10$ nuclei in the Dietrich and Berman compilation. Moreover, it provides an explanation for the larger polarization effects found in light nuclei with $10 \leqslant A \lesssim 20$. Additional work is needed to test this new equation and evaluate deviations from the hydrodynamic model. It is encouraging, though, that the curve nicely merges with the σ_{-2} data for $A \gtrsim 70$, in agreement with the dominant photoneutron cross sections for heavy nuclei.

Data evaluations of currently available photoproton and photoneutron cross sections remain to be done. The $\sigma(\gamma,p)$ data are scarce compared to the $\sigma(\gamma,n)$ data and extensive work is desirable throughout the nuclear chart. These new data are crucial to test the σ_{-2} sum rule and provide a means to remove the model dependency of $a_{\mathrm{sym}}(A)$, which, in turn, may lead to a better understanding of $3N$ forces, neutron stars and supernova cores.

Furthermore, this work has direct implications in (1) broadly used Coulomb-excitation codes such as GOSIA [36], where the polarization potential has to be modified with either Eq. (9), which requires a determination of κ for $A \lesssim 50$ nuclei, or Eq. (12), once its generality has been fully tested; and (2) shell model calculations of κ [37–39]. To date, both approaches have broadly regarded Levinger's empirical formula [Eq. (7)]. For clarity purposes, these implications will be presented in a separate manuscript.

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