Contemporary Problems of Teaching and Learning in Mathematics Education

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Abstract: *The way mathematics teaching and learning activities are presented to learners can make them hate or like the subject. The question of accomplishing the mathematics education of the learner from primary to post-secondary school levels is one which necessarily tasks, not only the teacher's stock of mathematical knowledge but also his skill, his method of approach and finally his handling of the processes of feedback mechanisms. This paper outlines some of the fundamental problems confronting mathematics teachers and learners, and by means of previously published work, it proposed some useful suggestions which have gained support through classroom implementation.*

Keywords: Mathematics, Learners, Teachers, Problems, Teaching methods

Introduction

Concerns about problems of mathematics education at primary, secondary and post-secondary school levels are not new. As far back as 1935, UNESCO started paying attention to these stages of studying mathematics education. Since then, various publications such as articles, books, reports and policies, etc. have been documented. In addition to this effort, other important systems like the Program for International Student Assessment (PISA), and the Trends in International Mathematics and Science Study (TIMSS), among others, have been monitoring the effectiveness of educational systems in various countries. For example, the results that were summarized by TIMSS on the *'Classroom Teaching Limited by Students Not Ready for Instruction'* showed a direct relationship between the degree that instruction was limited by students not ready for instruction and students' average achievement (Mullis, Martin, Foy, Kelly, & Fishbein, 2020, p. 420). An immediate consequence of this is that the instructional approach that teachers use to convey the mathematics curriculum to learners has significant implications for their learning.

In general, the subject of mathematics has been a scourge on learners (Beckmann, 2005; Carpenter, Moser, & Bebout, 1988; van der Walle, Karp, & Bay-Williams, 2010). One sorry aspect of this problem is that though it is recognised by all concerned for what it is, yet little is done to solve it.

Before most learners embark on the study of mathematics, even in its most elementary form, certain prejudices militating against the pursuit of the subject are generated in them. Some of these prejudices are inborn while others are due to external influences. Though a few learners possess innate abilities, a great many of them do not (Thompson, 1999). It is therefore left entirely to the external agents to, as it were, infuse into such learners whatever it takes to awaken the mathematical talent in the majority of learners, nurture some, and guide it on to full fruition so that its effective application may become a matter of course and routine. Our only hope is to start from scratch. Therefore, in this paper I consider the entire issue of mathematics education in three stages beginning with the primary level then on to the secondary, and finally ending up with the post-secondary level. In discussing each level, first of all, I identify the problems and then proceed, using specific illustrations and the extant literature to suggest basic methods of addressing them in the light of conditions peculiar to the level at which teachers operate.

Primary mathematics education

The problems that beset mathematics education in primary schools are many and varied. In the paragraphs that follow, I discuss some of these problems in the light of the approach outlined above. These problems are as follow:

Problem (1): The inability of many learners to grasp the concept of numbers early enough in their development.

It is true that there is much to be said for the theory of 'late developers', but it is also true that not only is it a fact that nothing is lost when development comes early, but also it is clear that the chances of improving on such development are good (Wright, 1994). On the other hand, it is also true that late developers may lose precious time during which fuller appreciation of what is learned could have come their way. The tackling of this problem must not wait until after the learner has passed through primary school, not even until he has matured mathematically on his own. It is essential to help him as much as possible so that this problem can be solved within the shortest time. The question that arises now is: how do we proceed to handle the issue? Here we have to be constantly aware of the fact that frequent exposure to ideas in general makes for better attention to and assimilation of the ideas. We must therefore expose the learners to situations and surroundings which offer them ample opportunities with the idea of numbers (Fuson et al., 1997; Wright, Martand, & Stafford, 2006). When it comes to the question of imparting the concept of numbers to learners and we want learners to understand the message which numbers bring, then the heuristic approach is very apt (Carpenter, Hiebert, & Moser, 1983; Gravemeijer & Stephan, 2002). Let the learners find out the concepts for themselves as the teacher guides them. In the process, even the necessary vocabulary becomes acquired in a bi-lingual situation. An instance of this approach which has gained support through classroom implementation in the work of Beckmann (2005), Gravemeijer and Stephan (2002), and Wright et al. (2006), among others are:

i) The teacher lays a collection of items on the table before the learners. These may be oranges, small rubber balls, Ping-Pong balls, or lemon fruit, etc.

ii) He first considers items of the same kind and distributes a few to each learner, or in fact asks them to bring their own collections from home which they must now display on their desks (see for example, Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

iii) The teacher now picks up one ball and shows to the learners saying: ‘one ball.’ He asks them to do likewise. He continues that with the other items. The learner now begins to see that though there are many items involved, a single instance from each collection has associated with it the idea of ‘one-ness.’

iv) Next the teacher picks up two items of the same kind at a time and proceeds as before: ‘Two balls’, ‘Two oranges’, etc. This makes the idea of ‘two-ness’ register on the minds of the young learners and so on (see for example, Anghileri, 2006).

v) The teacher now moves on to picking up one ball and one orange together, etc. The learners will have little or no difficulty in translating the idea of two-ness gained earlier to these two items. In this way, the concept of numbers becomes ingrained in the learners, in particular, late developers.

It does not matter in which single language a lesson in the format described above is carried out, the learner comes out in the end learning something about the concept of numbers (De Corte, 1995).

Problem (2): Improper introduction to the basic operations of arithmetic

The basic operations of arithmetic to which reference is made here are those of addition, subtraction, multiplication, and division. The challenge here, as with the concept of numbers, is an early comprehension and appreciation of what the concepts being learned are. As far as application is concerned, and as far as arithmetic as a unit of knowledge is concerned, helping learners in developing an understanding of the basic operations of arithmetic through everyday interactions will enhance the learner’s seamless and easier transference of one arithmetical concept to another (Wright et al., 2006). In the literature (Carpenter et al., 1989; Klein, Beishuizen, & Treffers, 1998; Mullis et al., 2020) it is clear that when one concept has not been fully grasped, the imposition of another concept, and yet another on so shaky a foundation leads eventually to a situation in which the whole system of mathematical knowledge becomes a bundle of confusion for the learner. The end result is frustration and aversion for mathematics (De Corte, 1995). Let us now look at the basic arithmetic operations in the light of the preceding argument. After gaining a good insight into the concept of numbers most learners are in a good position to follow this up with a successful handling of the basic operations of arithmetic (Klein et al., 1998). It is only left to the teacher to adopt the proper approach. According to Fuson et al. (1997), the concept of numbers must now be associated with concrete terms instead of dealing with numbers in the abstract. Ask a learner who has a good idea of numbers, how much he will have if his mother gave him three cents and then his father follows with two more cents. Without much effort he will tell you that he now has five cents. On the other hand, let the teacher just slap down on the whiteboard the problem '3 + 2 = ?,' chances are that a great many learners who are just beginning these arithmetic computations may not be able to supply the correct answer (Carpenter et al., 1983; Wright et al., 2006).

Before we go on, it will be useful to consider a rather famous mathematical proposition as an example. Suppose a teacher wants to test the proposition '5 + 4 = 9.' He takes five microbes and puts them on a slide, then takes four more and puts these on the slide also. And ask the learners to count them. Suppose the learners found/counted ten microbes on the slide, not nine. They might conclude that there was a microbe on the slide to start with, or that they miscounted, or that one of the microbes was divided into two. But under no circumstances will they conclude that '5 + 4' does not equal to 9! (Kemeny, 1959, p.17). Again, this is because the problem is treated in the abstract. What goes for addition also goes for subtraction. Once again, a question is put to the learner, 'your mother gives you five cents and you give one of the five cents to your sister, how many cents do you have left?' He may invariably answer, 'four cents' with little or no effort. In Humpty Dumpty's problem, the argument is somewhat as follows: "– 1" means going from a number to the previous one, "=" means being the same number, and hence the proposition asserts that the number 4 is the number right before 5, this is true, because that is how we named our numbers (Kemeny, 1959, p. 21). But let the problem be set as '5 – 1 = ?', and the learner is asked to show on paper how '4' is an answer, then he sees numbers in the abstract and develops cold feet (Carpenter et al., 1989; De Corte et al., 1994) .

Again, by seeing a number of cent arranged in groups which form units, e.g. in groups of three, the learner will soon learn that the total number of cent in two such groups is six, and in three such groups, nine, and so on. In other words he begins to see that the concept of addition which he had acquired can be extended to the concept of multiplicity. In this way multiplication begins to make sense to him (Kouba & Franklin, 1993). No amount of exercises like 3 × 2 = 6, 2 × 4 = 8, all dealing with numbers in this abstract manner could have achieved the same result within the same time as the practical approach of linking up the numbers with the operations (Lampert, 1986). In the case of division, which presents the greatest difficulty to learners at this level, again a down-to-earth approach is best. A typical learner knows, even if it be intuitively, that when he and his brother are given six oranges to share equally between themselves, he will have three oranges and his brother will also have three oranges after the division. But once more the problem 6 ÷ 2 =? creates a momentary black-out in the minds of many learners (Kouba & Franklin, 1993). We can thus draw the conclusion that the basic operations of arithmetic ought to be introduced to the beginning learner within the context of concrete terms or materials before attempts at abstraction can be made (Verschaffel, De Corte, & Lasure, 1994).

Problem (3): Heavy reliance on memory work in dealing with fundamental arithmetic processes

The emphasis on this problem is on the word 'heavy'. Traditionally, learners in our schools are made to commit certain bodies of mathematical knowledge and certain mathematical relations to memory. I refer for instance to the multiplication tables and to such other tables as those of weights and other measures. There could be some acceptable argument for memorizing these materials but certainly we must face reality in these matters. To memorise the multiplication tables and a few relations may be wholesome enough but there is a great need to associate such memory work with practical situations. While memorising relations for instance, learners should be exposed to practical situations which enable them to appreciate what they have been memorizing. There are for instance metre rules containing units of the millimetre, the centimetre, the decimetre, and finally the metre itself. A direct confrontation with these lengths is essential to make the lesson meaningful. Merely rattling away as my little niece of nine does: 'Ten millimetres make one centimetre' and so on, does little if any good at all. When I bring a ruler on which these units are inscribed and show them to her, she wants to ignore my effort emphasising to me that the teacher says they must know it by heart. In this situation, there is clearly something missing. The very notion of length seems to be absent as far as the child is concerned. This is a basic problem that must be cleared before the relations between units of lengths are tackled. In the absence of this kind of approach, terms such as centimetres, kilometres, kilograms, second, minutes, hours, etc. remain illusory to the learner for a long time (Anghileri, 2006; De Corte et al., 1994).

Problem (4): Lack of practice

We cannot afford to neglect the ancient aphorism 'practice makes perfect.' There is to be noticed in various countries much apathy in this matter of improving the standard of mathematics education (Banwell, Saunders, & Tahta, 1972; UNESCO, 2020). This arises from the attitudes of learners, teachers and educational authorities. Let us consider these in turn.

(a) **Attitude of learners:** Frustrated learners are difficult not only to encourage but also to control. Too many learners get frustrated by the way teachers handle the subject matter (Mullis et al., 2020). Some teachers are in the habit of intimidating their learners. This is sometimes very pronounced in the case of mathematics lessons. One hears teachers making statements like: 'Mathematics is very hard. Only very few people usually understand it', if you do not sit up, there is no mercy, you will fail.' The pity of the whole situation is that the fault may not necessarily arise from the learner, but from some of the stumbling blocks identified earlier. Invariably the learners develop: (1) hatred for arithmetic and numbers in general, and (2) hatred for the teacher involved. It is thus not a surprise that they turn rebellious, as it were, whenever they are confronted with mathematics (Carpenter et al., 1983; van der Walle et al., 2010).

(b) Attitude of teachers and government education authorities: The teaching profession in many countries has ceased to be a noble profession and has instead, become a 'stepping-stone' kind of employment. Too many teachers are itinerant workers, unhappy and dejected

(UNESCO, 2020). They move, and quite logically non-teachers who must subsist, if not exist as far as the battle for life is concerned fill the numerous vacancies that teachers create as they move away. Government educational authorities have always been concerned with this and other related problems. In many countries financial resources are limited and cannot do enough for teachers and education in general because of other commitments in these societies to which attention must be given, and this leads to a kind of vicious circle between government and teachers. Government cannot help teachers beyond a certain limit, and teachers on their part cannot help the government discharge its educational responsibilities beyond the level to which they can bear their crosses (Iwuanyanwu, 2019).

Problem (5): The Problem of adequate textbooks and materials

No one who goes to achieve anything can do so without adequate means. One of the most important means of achieving success in the study of mathematics in primary school, as in other levels of school, lies in the direction of books and availability of relevant materials (Stigler, Fuson, Ham, & Kim, 1986). It is most gratifying to note that in this day of instant and global information access, digital books, software programs, and apps on mathematics are springing up across the world. Some programs aimed at helping learners to develop the computing ability of addition and subtraction include digital mathematical games, e.g., Cross Number Puzzle game for practicing arithmetic expressions by Chen, Looi, Lin, Shao, and Chan (2012).This has moved the burden of the work on teachers who must produce handouts and notes for learners where textbooks are not available. However, due to lack of access to internet resources and/or unavailability of network infrastructures in some developing countries, the circulation of digital materials is painfully low. What has been said for books also applies to materials and equipment which go into the teaching and learning process. In this connection however, while conceding the fact that teachers may be relied upon to improvise at crucial moments, such moments must not assume a high frequency of occurrence. Educational authorities must play their part effectively by making adequate provisions in this connection. Let us now proceed to consider the case of secondary mathematics education.

Secondary mathematics education

The main argument in our discussion on primary mathematics education rests on the avoidance of leading the learner to frustration, aversion, and in fact hatred not only of mathematics as a subject but also, in some cases at least, of the very teacher who handles mathematics. If the teacher and all concerned must avoid this line of action, then it is clear that an all-out attempt to arouse the learner's interest in the subject is the most logical line of action to follow. As a way to create and sustain learners' interest in, participation in, mathematics, some countries have invested in teacher development training on how to use innovative teaching methods, digital resources and tools in mathematics teaching. However, the extent to which this happens in practice varies, owing to a lack of computers, teachers' critical attitude, or their unwillingness to change traditional habits (Kearney, 2010, p. 6). In fact, common sense brings this stand out clearly for when we are faced with any difficult problem, we become uneasy and tensed up. This generates in us a desire to solve the problem so that we may be released from such tension and therefore so that satisfaction may come our way. But to desire something is to have developed an interest in the object of desire, hence we cannot successfully equate 'difficulty' with 'lack of interest.' As Hartung (1953) put it:

> Genuine interest in mathematics probably depends upon the problem solving aspect of the subject. Problems once recognised or sensed, leave an individual in a state of perplexity, uneasiness or tension until they are solved. When a solution has been found, tension-reduction and satisfaction results. (p. 51)

There is therefore agreement that if mathematics is properly taught, it presents the learner with an abundance of problems, and it also provides him with certain general modes of thought and a supply of techniques which enable him to attack these problems successfully (Anghileri, 2006; Carpenter et al., 1989; Kouba & Franklin, 1993; Mullis et al., 2020). With each successful solution he receives a dividend of satisfaction – "he feels good when he gets the answer" (Hartung, 1953, p. 52). As a result, he seeks more experiences of the same kind. It is the search for more experiences described by Hartung that expresses in unmistakable interest which the learner develops in mathematics. The question now left to us to answer is how we as teachers can help secondary school learners develop such an interest. We note in passing that our problem calls for a partially curative and partially preventive approach – curative because for a great many learners we must start by destroying the unsavoury image of mathematics painted for them by teachers/others and conditions before this stage, and preventive because we must now seek basic approaches not only to generate interest but also to sustain interest in the subject. Among curative approaches that have been advanced so far include such activities as (a) the awarding of prizes in mathematics to deserving learners, (b) the encouragement and guidance of learner mathematics clubs and societies, (c) the organisation of state-wide and nation-wide annual contests in mathematics, (d) the formation and support of active mathematical organizations reflecting the progress of the subject in the entire country, (e) the provision of good teachers of mathematics and (f) the provision of good textbooks and materials in the subject (see for example, Martin & Dowson, 2009; Middleton, & Spanias, 1999).

Let us now proceed to consider an instance of the preventive approach to the problems of generating and sustaining interest in mathematics in our secondary schools. The onus here rests in the first instance with the teacher. He is the one who first confronts new learners of mathematics. If he continues with the type of poor approaches that we criticized at the primary level, then these new learners are half-doomed even before they start their actual studies in the subject (Stipek, Givven, Salmon, & MacGyvers, 2001). A very glaring instance here is the usual presentation of so basic and yet so important a topic as 'The rule of signs in algebra'. A learner asks: 'Why is it that $(-2) \times (-3) = (+6)$?' The teacher replies; 'Because minus × minus = plus, according to the rule of signs.' There is nothing that can be so baffling to a new learner as such unexplained concepts. And there are so many such concepts in mathematics. Invariably most of the learners end up memorising these concepts and this is not studying mathematics. Naughty problems such as that posed by the so-called Rule of Signs can be explained to the delight of learners via such simple devices as the Number Line which removes a good measure if not all of the abstraction involved. Let us consider the number line AB shown below.

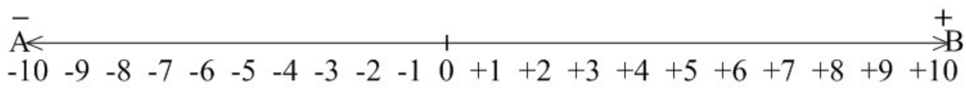

Figure 1 Number line

We shall make a number of movements on this line, and 0 is the starting point of all such movements. It is therefore called the origin. Multiplication always involves two numbers; the first is the multiplicand while the second is the multiplier. In general if the sign of the multiplicand is positive, we stand at the origin and face the positive direction B, as shown in Figure 1. If it is negative, we also stand at the origin but face the negative direction A. Next, we consider the sign of the multiplier. If it is positive, we continue to face whichever direction we are facing. If it is negative, however, we turn around and face the opposite direction. Thus in the problem (-2) × (-3), the multiplicand is negative so we stand at 0 and face the negative direction A. But the multiplier is also negative so we turn around and now face the opposite direction which in this case is the positive direction B which is our final direction. Now (2) × (3) = 6. But our final direction is +, therefore (-2) × (-3) = (+6). According to Klein et al. (1998) this kind of approach makes a good measure of concretization of an otherwise abstract concept. This is the type of approach on the part of the teacher which leads to the generation and sustenance of interest in the learner while s/he studies mathematics (Martin & Dowson, 2009; Stipek et al., 2001). It is part and parcel of the basic approach.

At this point I must interject my contention with a true-life experience. It took me some fifteen years of schooling to get an inkling of what mathematics is all about. My high school years of studying mathematics bribed me into believing that mathematical propositions are self-evident truths. There was no choice left. However, it bothered me to know that it took my mathematics teacher weeks of hard work to convince us how self-evident it was that the base angles of an isosceles triangle are equal. Here is another axiom that teachers usually feel handicapped to explain to learners which I think you are familiar with: *'Given a line, and a point not on the line, there is just one line through the point parallel to the line.'* One of the consequences of this axiom together with the other axioms is that there seems to be no practical method for testing this directly. No effort has succeeded in proving this axiom. So, before you find yourself spending the rest of your life on this task, someone should warn you that all the other axioms could be checked by making small diagrams, but this troublesome axiom stated that the two lines never meet, for which there is no practical test. When this kind of experience is considered at the level of the student starting out with the study of mathematics, his/her bewilderment can only be imagined and not described

(Hartung, 1953). Let us now go on to examine the final stage of the general problem of mathematics education at post-secondary level.

Post-secondary mathematics education

The problems of mathematics education in post-secondary institutions are not as disturbing as those in the primary and secondary levels. Although some students in post-secondary education are put off by mathematics and are unable or unwilling to reason, and in certain circumstances, they merely have rudimentary memorization and copying skills. Others are ready, able, and motivated to do mathematics. However, the scarcity (in terms of numbers) of mathematics graduates is a major source of worry at this level of education. But this is clearly due to the foundation upon which mathematics education in many countries has been built (De Corte et al., 1994). Because of the problems already outlined before this stage, only a few who have vindicated the truth in the statement ‘survival of the fittest’, remain to pursue mathematics at the post-secondary level. To a great extent, problems bearing on the teacher’s approach in his handling of mathematics at post-secondary institutions are eventually resolved by the students whether this be done among themselves or with the aid of superior wranglers around the campus. However, there are usually certain topics in higher education mathematics which do not come home readily to students. It often requires much more than the normal dose of lectures to pound such topics home to students. But such an attempt jeopardises the study of other topics, which should not be the case. We are therefore forced to accept the conclusion that ways and means should be devised to attack this problem. One such approach would be the use of practical illustrations to handle such ‘stubborn’ topics. The following instance dealing with the idea of limits will clarify the point being made here. The bewilderment which grips a beginning student in the study of limits when confronted with a statement like: Let $f(x)$ be defined for all $x \neq a$ approaches over an open interval containing a. Let L be a real number. Then $\lim_{x \to a} f(x) = L$ if, for every $\varepsilon > 0$, there exists a $\delta > 0$, such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$ is better experienced than described.

Now let us consider the following extract:

“Suppose we are manufacturers of nails and that a certain order specifics nails of exactly 3 cm in length. If we now adjust our manufacturing processes and each nail we turn out thereafter is exactly 3cm long, then we shall be achieving perfection. But it does turn out that in such practical situations perfection is never achieved as a matter of rule, that is, if at all it is achieved. Wear and tear on the machine as well as human short-comings lead to variations in the lengths of the nails produced, no matter how small these variations may be. A prospect buyer realising these imperfections may then specify in his order that though he wanted nails exactly 3cm long, he would accept nails not less than or greater than 3cm by 0.1cm. He thus assigns a margin of error in the lengths of the nails produced. In other words, nails whose lengths fall outside this margin would be unacceptable to him. The margin of error is designated by the letter (ε). Thus here $\varepsilon = 0.1cm$. Clearly then if the length of a nail has values as 3.1cm, 3.09cm, 3.01cm, 3.002cm, or at the other end -3.001, -3.02, -3.06, and -3.1, the customer will be satisfied. If however the length has values such as 3.11, 3.12, 3.2, all of which are greater than 3.1, or values such as – 3.104, -3.11, -3.2, all of which are less than – 3.1, then the customer will refuse such nails.

Now let the aggregate of factors due to men, machines, and materials, etc. which go to determine the length of a nail be represented as x. In other words, the length of a nail is a function of x. Thus if y is the length of a nail, then we write $y = f(x)$. The difference between $f(x)$ and the perfect length 3cm of a nail can be written as $|f(x) - 3|$. The absolute value signs show that $f(x)$ may be slightly greater or slightly less than 3cm. This difference must be less than epsilon (ε) for the nail to be accepted by the customer, that is we require that $|f(x) - 3| < \varepsilon$. Now let the value of x which gives a nail of exact length 3cm be b. Then a deviation from this value can be $b - x$ or $x - b$, hence generally the deviation can be represented by $|x - b|$. This deviation must be small to ensure that $|f(x) - 3| < \varepsilon$. This condition is written as $|x - b| < \delta$, where δ is small. Hence, we have the statement: $|f(x) - 3| < \varepsilon$ whenever $0 < |x - b| < \delta$, which is the same as the statement that bewildered the beginning student in the study of limits.

Closing thoughts

What has emerged in this essay is that in each of the three stages of teaching mathematics to learners (i.e., primary, secondary and post-secondary school levels), teachers' modes of instruction and practice need to change. Several factors highlighted in this paper help explain why a shift from presenting numeracy and arithmetic concepts in abstract forms to learners also need to change to more practical examples. However, at some point in their mathematical education learners should switch from practical examples to abstract reasoning in order to succeed in algebra and beyond, as well as to like mathematics. The significance of the abstract approach to teaching mathematics in scenarios involving 4-D spaces, for example, cannot be geometrically represented (visualized) in our 3-D world since they are defined in the abstract forms. In this sense, 4-Dimensional spaces, despite their formal description, have a wide range of applications, including "The Quaternions," which are used to model Einstein's "General Theory of Relativity". Finally, given the seemingly conflicting efforts to reform mathematics instruction (and foster learners' interest and participation in the subject) teachers who do not learn to reflect on their teaching are unlikely to understand the rationale for certain practices and are therefore less able to select and interpret instructional practices that support motivation and learning (Turner, Warzon, & Christensen, 2011). Ultimately, once the bewildering moments experienced by mathematics learners at the early stages of their schooling are taken care of, the solution of the problems of mathematics education shall have been halfway accomplished.

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