Capture zone simulation for boreholes located in fractured dykes using the linesink concept

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Abstract

Delineation of capture zones for groundwater source protection is normally performed by using numerical codes which are based on the porous medium flow equation. However, boreholes are often sited in or along permeable dykes or single fracture zones through which aquifers are drained. It is very important to take into account dyke-influenced aquifers. This paper makes use of Linesink to simulate permeable dyke or fractured zones and utilises the pathline distribution to delineate the capture zones. Conditions when the influence of a fractured dyke can be considered negligible are also discussed through comparison with stagnation point in a uniform flow field. The approach may be sufficient to illustrate protection zoning requirements when dyke aquifers are considered.

Introduction

Studies on capture zones for simple flow conditions in uniform aquifers were performed by Todd (1980), Almendindger (1994) and others. For more complex flow situations where boundaries are considered, borehole capture zones or catchments may be delineated by using semi-analytical models (Nelson, 1978a,b; Keely and Tsang, 1983; Javandel and Tsang, 1986; Lerner, 1990 & 1992; Blandford and Huyakorn, 1991 and Kinzelbach et al., 1992).

The existing semi-analytical models provide a powerful tool to understand the capture zone concept and to acquire general ideas about borehole or wellhead protection zoning before embarking on a site-specific study of groundwater protection. However, these models do not account for the capture zone of a draining fracture. In South Africa, boreholes are often sited in highly fractured dykes for good water supplies. Strack (1989) and Haitjema (1995) presented the concept of the linesink which is utilised here to simulate a permeable dyke or fracture zone for delineating the capture zone in dyke aquifers.

A borehole protection area can be defined as the controlled area surrounding a production borehole (or wellfield). Demarcation of such controlled areas where certain activities of land use are prohibited would prevent contaminatants from reaching the borehole. It may consist of a capture zone as well as a borehole catchment. The latter, also referred to as the zone of contributing water (ZOC) (Todd, 1980; Reilly and Pollock, 1993), is the limiting case of the capture zone at t where $t \to \infty$. The borehole catchment may be interpreted as the projection on ground surface of a 3D aquifer volume which would contribute water to the borehole under steady-state flow and pumping conditions. Inside the catchment water would flow towards the borehole whereas water outside would flow away from the borehole. The delineation of the protection area is often based on assumption of the averaged steady-state flow. Assuming A (L^2) is the area of the catchment, P (LT^{-1}) a uniform rainfall recharge and Q (L^3T^{-1}) the averaged pumping rate from a borehole of interest, then the following relation holds:

$$AP = Q \tag{1}$$

Eq. (1) tells us that the borehole catchment size A can be calculated if P and Q are known. However, Eq. (1) gives neither a physical location of the catchment with respect to the borehole, nor does it provide hydrogeological conditions like type of aquifer, etc. It merely provides a water balance with as many geometrical distributions as possible (illustrated in Fig. 1).

Under steady-state conditions, groundwater streamlines coincide with fluid pathlines. If dispersion is negligible, we may use pathline equations to track pollutant movements in aquifers. To demarcate either a capture zone or a borehole catchment under the steady-state, we utilise discharge potential to derive the pathline equations. Assuming that an aquifer thickness is more or less uniform, the pathline distribution under certain hydrogeological settings is investigated in an x, y plane.

Theory of capture zone simulation using linesink concept

For the sake of simplicity, we assume that the natural hydraulic gradient can be neglected. This is often encountered in aquifers interrupted by vertical dykes. If the gradient is assumed to be zero, a pumping borehole in a uniform aquifer will cause a circular shape of the cone of depression with radius r, which can be directly calculated from the formula: $r = \sqrt{(A/\pi)}$, where A is obtained from the water balance. However, the focus is on the following cases.

Discharge potential for fracture zone

Based on Strack (1989) and Haitjema (1995), a linesink can be defined as a mathematical sink line with a finite length. If a pumping hole is located in a fracture zone, the fracture can be regarded as an extension of the borehole. It is noticed that the behaviour of a pumping hole located in the fracture zone may be similar to that of a linesink. A fracture zone can be simulated by the linesink. Based on the complex potential for the linesink element with length L (Strack, 1989), the discharge potential may be written as follows:

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Received 30 July 2001; accepted in revised form 14 December 2001.

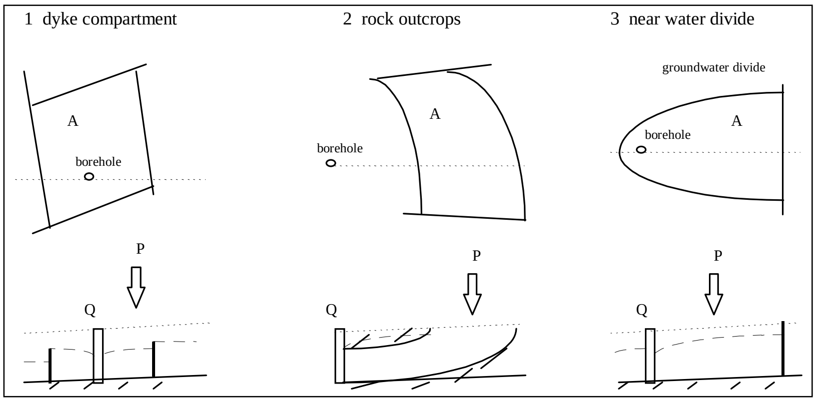

Figure 1
Possible locations of borehole catchment

$$\Phi(x,y,L)=\frac{Q}{2\pi}\left\{\left(\frac{x+\frac{L}{2}}{L}\right)\ell n\left(\left(\frac{2(x+\frac{L}{2})}{L}\right)^2+\left(\frac{2y}{L}\right)^2\right)^{\frac{1}{2}}-\left(\frac{x-\frac{L}{2}}{L}\right)\ell n\left(\left(\frac{2(x-\frac{L}{2})}{L}\right)^2+\left(\frac{2y}{L}\right)^2\right)^{\frac{1}{2}}-\frac{y}{L}\left(\theta_1(x,y)-\theta_2(x,y)\right)+\ell n\frac{L}{2}-1\right\} \tag{2a}$$

where:

$\theta_1(x,y) = arctan(y/(x+L/2))$ or $\pi + \theta_1(x,y)$ if $\theta_1(x,y) \le 0$
$\theta_2(x,y) = arctan(y/(x-L/2))$ or $\pi + \theta_2(x,y)$ if $\theta_2(x,y) \le 0$
$\rho_1 = ((x-L/2)^2+y^2)^{1/2}$, $\rho_2 = ((x+L/2)^2+y^2)^{1/2}$

Derivation of velocity expressions

Applying Darcy's law to Eq. (2a), i.e. $V_x = -(1/Hn)d\Phi/dx$ and $V_y = -(1/Hn)d\Phi/dy$, note that H and n stand for aquifer thickness and effective porosity, respectively. We may derive the expression for the velocity field for the linesink:

$$V_{lsx}(x,y,L)=-\frac{Q}{2\pi Hn}\left\{\frac{1}{L}\ell n\left(\frac{\rho_2}{\rho_1}\right)+\frac{x}{L}\left(\left(\frac{x+\frac{L}{2}}{\rho_2^{\,2}}\right)-\left(\frac{x-\frac{L}{2}}{\rho_1^{\,2}}\right)\right)+\frac{1}{2}\left(\left(\frac{x+\frac{L}{2}}{\rho_2^{\,2}}\right)+\left(\frac{x-\frac{L}{2}}{\rho_1^{\,2}}\right)\right)+\frac{y}{L}\left(\left(\frac{y}{\rho_2^{\,2}}\right)-\left(\frac{y}{\rho_1^{\,2}}\right)\right)\right\} \tag{2b}$$

$$V_{lsy}(x,y,L)=-\frac{Q}{2\pi Hn}\left\{\frac{x}{L}\left(\left(\frac{y}{\rho_2^{\,2}}\right)-\left(\frac{y}{\rho_1^{\,2}}\right)\right)+\frac{1}{2}\left(\left(\frac{y}{\rho_2^{\,2}}\right)+\left(\frac{y}{\rho_1^{\,2}}\right)\right)-\frac{y}{L}\left(\left(\frac{x+\frac{L}{2}}{\rho_2^{\,2}}\right)-\left(\frac{x-\frac{L}{2}}{\rho_1^{\,2}}\right)\right)-\frac{1}{L}\left(\theta_1(x,y)-\theta_2(x,y)\right)\right\}$$

Therefore the pathline equations may be written as follows:

$$\frac{dx}{dt}=V_{lsx}(x,y,L) \qquad \frac{dy}{dt}=V_{lsy}(x,y,L) \tag{2c}$$

Integration of Eq. (2c) would give the pathline distribution, which may be used to backtrack isochrones or capture zones. However, such pathline equations are not analytically obtainable. Eq. (2c) has to be numerically solved using the fourth-order Runge-Kutta procedure.

Considering the impermeable boundary

Todd's method (Todd, 1980) is acceptable for the infinite aquifer assumption. For a dyke-influenced aquifer, Todd's method must be examined. We still use the above dyke aquifer as the example to discuss catchment distribution of the fracture zone. In an x, y plane, the impermeable dyke is located at $x = - d$ and the centre of the fracture with length L at the origin $(0, 0)$. The regional flow due to uniform rainfall recharge P is directed towards $x = \infty$. Three cases are discussed:

(1) a catchment for a single borehole;
(2) a catchmnet for a horizontal fracture; and
(3) a catchment for a vertical fracture.

However, below we only derive the pathline equation for Case (2). The others may be derived in similar fashion. The identical pathline equation for Case (1) is also derived by Kinzelbach et al. (1992).

Following the principle of superposition, the combined velocity for Case (2) is the summation of three components: flow due to rainfall recharge P, linesink V_{ls} and its image V_i. The corresponding pathline equation may be written as follows:

$$\frac{dx}{dt}=P\left(\frac{x+d}{Hn}\right)+V_{lsx}(x,y,L)+V_{ix}(x,y,L,d) \qquad \frac{dy}{dt}=V_{lsy}(x,y,L)+V_{iy}(x,y,L,d) \tag{3a}$$

where the linesink V_{ls} is given by Eq. (2b) and its image V_i is given below:

$$V_{ix}(x,y,L,d)=V_{lsx}(x+2d,y,L) \qquad V_{iy}(x,y,L,d)=V_{lsy}(x+2d,y,L) \tag{3b}$$

In order to present a compact form of the catchment, dimensionless parameters are introduced. They are:

$$v^*=\frac{VH}{Pd},\ Q^*=\frac{Q}{2\pi Pd^2},\ x^*=\frac{x}{d},\ y^*=\frac{y}{d},\ L^*=\frac{L}{d},\ t^*=\frac{Pt}{Hn} \tag{3c}$$

Substitution of Eq. (3c) in Eq. (3a) leads to the dimensionless form of the pathline equation (Eq. (3a)) as follows:

$$\frac{dx^*}{dt^*}=\frac{x^*+1}{n}-\frac{Q^*}{n}\left\{\frac{1}{L^*}\ell n\left(\frac{\rho_2^*}{\rho_1^*}\right)+\frac{x^*}{L^*}\left(\frac{x^*+\frac{L^*}{2}}{\rho_2^{*2}}-\frac{x^*-\frac{L^*}{2}}{\rho_1^{*2}}\right)+\frac{1}{2}\left(\frac{x^*+\frac{L^*}{2}}{\rho_2^{*2}}+\frac{x^*-\frac{L^*}{2}}{\rho_1^{*2}}\right)+\frac{y^*}{L^*}\left(\frac{y^*}{\rho_2^{*2}}-\frac{y^*}{\rho_1^{*2}}\right)+\frac{1}{L^*}\ell n\left(\frac{\rho_{2i}^*}{\rho_{1i}^*}\right)+\frac{x^*+2}{L^*}\left(\frac{x^*+\frac{L^*}{2}+1}{\rho_{2i}^{*2}}-\frac{x^*-\frac{L^*}{2}+1}{\rho_{1i}^{*2}}\right)+\frac{1}{2}\left(\frac{x^*+\frac{L^*}{2}+1}{\rho_{2i}^{*2}}+\frac{x^*-\frac{L^*}{2}+1}{\rho_{1i}^{*2}}\right)+\frac{y^*}{L^*}\left(\frac{y^*}{\rho_{2i}^{*2}}-\frac{y^*}{\rho_{1i}^{*2}}\right)\right\} \tag{3d}$$

and

$$\frac{dy^*}{dt^*} = -\frac{Q^*}{n}\left\{ \begin{aligned} &\frac{x^*}{L^*}\left(\frac{y^*}{\rho_2{}^{*2}} - \frac{y^*}{\rho_1{}^{*2}}\right) + \frac{1}{2}\left(\frac{y^*}{\rho_2{}^{*2}} + \frac{y^*}{\rho_1{}^{*2}}\right) - \frac{y^*}{L^*}\left(\frac{x^*+\frac{L^*}{2}}{\rho_2{}^{*2}} - \frac{x^*-\frac{L^*}{2}}{\rho_1{}^{*2}}\right) - \frac{1}{L^*}\left(\theta_1^*(x^*,y^*) - \theta_2^*(x^*,y^*)\right) + \\ &\frac{x^*+2}{L^*}\left(\frac{y^*}{\rho_{2i}{}^{*2}} - \frac{y^*}{\rho_{1i}{}^{*2}}\right) + \frac{1}{2}\left(\frac{y^*}{\rho_{2i}{}^{*2}} + \frac{y^*}{\rho_{1i}{}^{*2}}\right) - \frac{y^*}{L^*}\left(\frac{x^*+\frac{L^*}{2}+2}{\rho_{2i}{}^{*2}} - \frac{x^*-\frac{L^*}{2}+2}{\rho_{1i}{}^{*2}}\right) - \frac{1}{L^*}\left(\theta_1^*(x+2^*,y^*) - \theta_2^*(x^*+2,y^*)\right) \end{aligned} \right\} \quad (3e)$$

where:

$$\rho_1{}^* = \sqrt{\left(x^*-\tfrac{L^*}{2}\right)^2 + y^{*2}},\ \rho_2{}^* = \sqrt{\left(x^*+\tfrac{L^*}{2}\right)^2 + y^{*2}},\ \rho_{1i}{}^* = \sqrt{\left(x^*-\tfrac{L^*}{2}+2\right)^2 + y^{*2}},\ \rho_{2i}{}^* = \sqrt{\left(x^*+\tfrac{L^*}{2}+2\right)^2 + y^{*2}},$$

$$\theta_1{}^*\left(x^*,y^*\right) = \arctan\frac{y^*}{x^*+\frac{1}{2}},\ \theta_2{}^*\left(x^*,y^*\right) = \arctan\frac{y^*}{x^*-\frac{1}{2}}$$

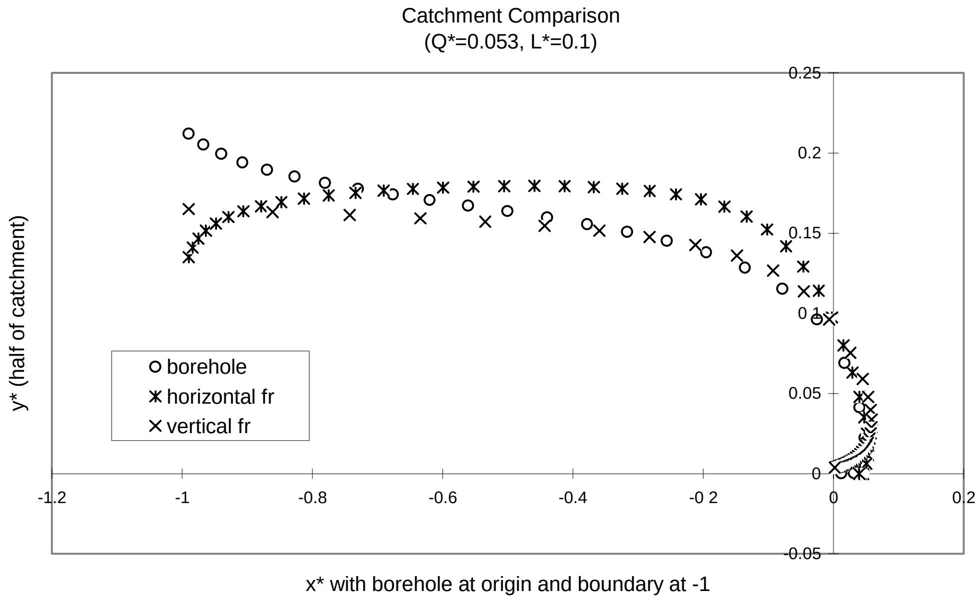

Figure 2
Comparison of three catchments

Based on Eqs. (3d) and (3e), the dimensionless catchment of the linesink element may be delineated. A comparison of three catchments is presented in Fig. 2. It can be seen that Todd's approximation is valid for a vertical fracture, i.e. a fracture parallel to the impermeable boundary. Dimensionless results for the shapes and dimensions of capture zones and catchments can be transposed to field situations. The dimensionless catchment in Fig. 2 would represent a typical hydrogeological setting of a Karoo aquifer in South Africa where an aquifer of 20 m thick with effective porosity of 1% and hydraulic conductivity of 20 m/d receiving uniform recharge of 4.7 cm/yr, pumping at a rate of 0.5 l/s from a vertical fracture of 100 m long located 1 000 m away from an impermeable boundary.

It is noted that the catchment of a horizontal fracture, a fracture normal to the impermeable boundary, would expand almost laterally on the fracture element side when Q^* increases.

Discussion

Comparison between linesink and point sink

The relationship between stagnation points between point sink and linesink under a uniform flow field may be expressed by:

$$\left(\frac{L}{x_s}\right) - Ln\left(\frac{x_{shfr}+\frac{L}{2}}{x_{shfr}-\frac{L}{2}}\right) = 0 \quad (4)$$

where X_s is the stagnation point position as used in Todd's approximation. The stagnation point X_{shfr} is due to a horizontal fracture with length L, Eq. (4) can be solved by iteration. It can be shown that $X_{shfr} \rightarrow X_s$ at $L \rightarrow 0$. In other words, if the fracture length L is small enough, X_{shfr} would be the same as X_s. This is illustrated by a straight line in Fig. 3. By replacing X_{shfr} with X_s in Eq. (4), we may determine the maximum allowable L_{max} value within which the fracture length may be negligible and be treated as a single borehole.

Simplified geometry of capture zone

The capture zone for a fracture zone is similar to an ellipse. It is noted that the capture zone of the fracture may be accurately represented by an ellipse when 0.02 m/d < L/t < 2 m/d. If L/t is greater than 2 m/d, the capture zone would show circular shape.

If a capture zone of a draining fracture is replaced by an ellipse, the area A of the capture zone equals ab where a and b are the lengths of the principal axes of the ellipse. For a fracture with length L, the a and b are the backtracking distance from a point ($L/2$, 0) along the positive x axis and the backtracking distance from a point (0, 0) along the positive y axis, respectively. The a and b are obtained by solving the above pathline equations using the fourth-order Runge-Kutta procedure.

Alternatively, the a and b may be estimated by solving Eq. (5) using iteration:

$$\frac{Qt}{Hn} = \pi\left\{a^2 - \left(\frac{L}{2}\right)^2 - \frac{2}{3}\left(\frac{L}{2}\right)^2 \ell n\frac{3a^2-\left(\frac{L}{2}\right)^2}{2\left(\frac{L}{2}\right)^2}\right\} \quad (5)$$

$$b = \frac{Qt}{\pi a H n}$$

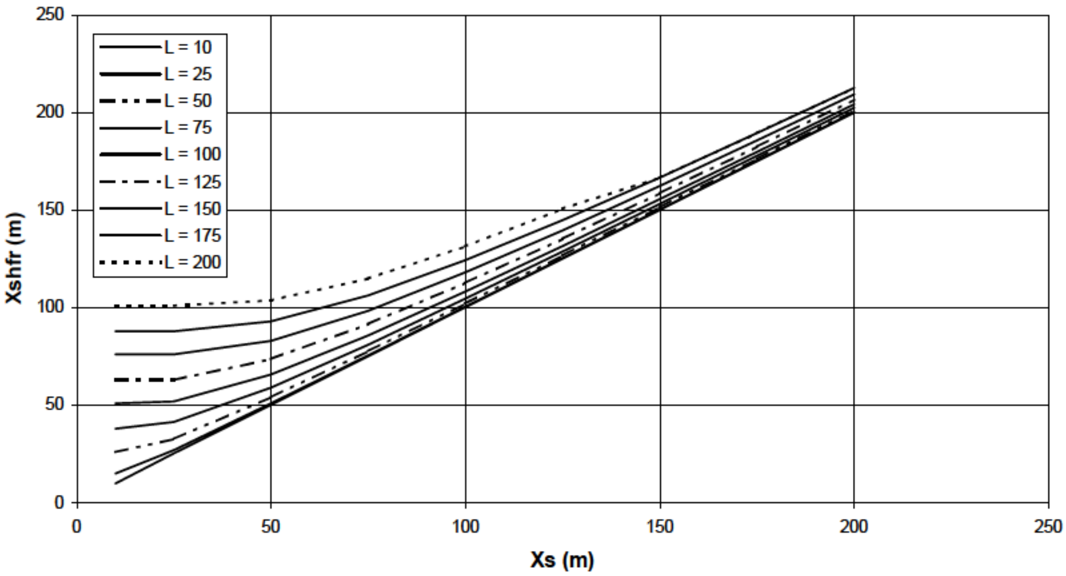

Figure 3
Relation of stagnation positions with line-sink length L (m)

Eq. (5), derived from the discharge potential of three boreholes with equal spacing and dewatering along a dyke, is an approximate pathline equation along the positive x axis for the fracture sink. It gives good results when $L < \sqrt{(16Qt/3\pi Hn)}$.

Once a and b are obtained, the co-ordinates of the ellipse can be calculated. However, implementation of such exact ellipsoidal geometry may not be necessary and rewarding in practice. Implementers from the local Water Boards or the Local Authorities would prefer some simpler geometry, for it would facilitate actual implementation processes in the field. Obvious alternatives are two simpler approaches:

- using an internal circle with area $A = \pi b^2$, resulting in an underprotection; and
- using an external rectangle with area $A = 4ab$, leading to an overprotection.

For convenient compromise and easy understanding, a combination of these simpler shapes is appealing. As seen in Fig. 4, an ellipsoidal area (πab) (solid line) may be replaced by A_{app} (outer broken line) consisting of a rectangle ($4bc$) plus two semi-circles ($\pi(a-c)^2)/2$ on either sides:

$$A_{app} = \frac{\pi}{2}(a-c)^2 + 4bc + \frac{\pi}{2}(a-c)^2 \qquad (6)$$

where:
$c = \sqrt{(a^2-b^2)}$

According to Eq. (6), $A_{app} \rightarrow \pi r^2$ at $L \rightarrow 0$, $c \rightarrow 0$ and a, $b \rightarrow r$; $A_{app} \rightarrow 4ab$ if $c \rightarrow a$. It is obvious that $\pi b^2 < A_{app} < 4ab$.

A discernible feature of this flow field is that the flow is symmetrical about either x axis or y axis and that central lines (x = 0 or y = 0) represent theoretical no-flow boundaries. This may lead to more applications of this approximation in situations like wellfield, and aquifers with impermeable boundary and anisotropic property.

Other consideration for application

Eq. (3a) makes it possible to backtrack pollutant movements in dyke aquifers. As an application example of the above theory, the delineation of borehole protection areas for dyke aquifers in South Africa is discussed.

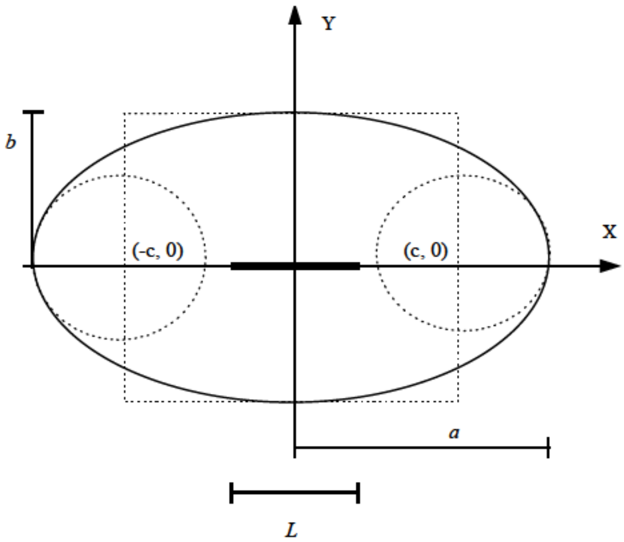

Figure 4
Approximation of ellipse

The important step is to carry out a water balance analysis, the purpose of which is to:

- independently examine variables like a pumping rate (Q), a travel time (t), aquifer thickness (H), effective porosity (n) and the rainfall recharge rate (P), which would help one conceptualise an aquifer being dealt with; and
- determine areal size of protection zone to be involved over a period of time, which would require economic considerations such as availability and sustainability of water resources and land use.

In certain parts of the country where aquifers are compartmentalised by vertical dykes, sizes of the compartments can be estimated from 1: 50 000 topographical maps. They may be compared with the calculated areas required for borehole protection. Based on results of the water balance, some important management decisions may be undertaken at this stage. For instance, a whole compartment will have to protected if the area estimated from the water balance is close to the actual size of the compartment or even slightly larger than it.

Conclusion

The fractured dyke can be regarded as an extension of the borehole. Flow behaviour of dyke aquifers can be simulated using the linesink concept. A capture zone for boreholes located in fractured dykes differs from that of boreholes in a uniform flow field. However the difference would diminish if the dyke length *L* were small. Based on discussions in this paper, a semi-analytical model could be constructed for conceptual modelling of capture zones in fractured rock aquifers. This provides a simple tool to understand protection-zoning requirements when dyke aquifers are considered.

Acknowledgement

The authors would like to thank E Braune of the Department of Water Affairs and Forestry for providing necessary support for the research on this subject.

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