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A re-assessment of the nearest neighbour alignment of the X-ray isophotes of galaxy clusters

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ABSTRACT

Alignment is defined as the tendency of the distribution of pointing angles between the major axes of clusters and their nearest neighbours to be more concentrated towards small values for small nearest neighbour distances, whereas the distribution is expected to be uniform over all angles at larger distances. Conflicting pronouncements on the reality of this effect have been published in the astronomy literature. A re-assessment of the evidence for alignment is presented, based on three recently published X-ray data sets. We find that whereas there is evidence for alignment, it is not as convincing as previously claimed. In particular, the scale to which the effect has been claimed to extend seems to have been severely overstated.

Key words: methods: statistical – galaxies: clusters: general – galaxies: formation.

1 INTRODUCTION

It is well known that many galaxies inhabit clusters, which themselves are organized into superclusters. There is some debate in the astronomy literature as to evidence for a tendency of cluster symmetry axes to be aligned with the direction to its nearest supercluster neighbour, provided the neighbour is close enough (see e.g. the brief literature review in Chambers, Melott & Miller 2000, hereafter CMM1). The point is of some interest, as proof of alignment may help to constrain supercluster formation theories (Ulmer, McMillan & Kowalski 1989, hereafter UMK). Colberg, Krughoff & Connolly (2005) used results from simulations by Kauffmann et al. (1999) to study the intercluster filamentary structure in a Λcold dark matter (ΛCDM) 'pancake' model. They concluded that cluster pairs closer than about 5 h^{-1} Mpc are almost always connected by filaments, but that the probability of a connection decreases at larger separations. If new clusters are preferentially accreted along filaments (e.g. Shandarin & Klypin 1984), and if cluster axes are preferentially aligned with infall directions (van Haarlem & van de Weygaert 1993), then the directional correlation mentioned in the previous paragraph may be expected. A similar recent simulation study showing the alignment of cluster major axis with nearest neighbour connecting line has been reported by Faltenbacher et al. (2005) (references to earlier work can be found in CMM1). We mention in passing that it may also be possible to accommodate alignment effects within hierarchical cosmogonies by invoking tidal interaction amongst clusters (see Salvador-Solé & Solanes 1993).

This paper is concerned with the re-evaluation of some of the statistical tests for alignment presented in three papers based on X-ray images of clusters of galaxies, namely UMK, CMM1 and Chambers, Melott & Miller (2002) (hereafter CMM2). The data analysed by these authors consisted of nearest neighbour distances and pointing angles for small collections of clusters. ('Pointing angle' is the smallest angle between the cluster major axis and the connecting line to the nearest neighbour cluster; pointing angles are defined to be in the interval $[0°, 90°]$.)

It is to be expected that any alignment would decline with increasing nearest neighbour distance d, i.e. in the mean the pointing angle should be small for small d; increase with increasing d and be close to an average of 45° for large nearest neighbour distances. This has the trappings of a regression problem, with d the independent variable and the pointing angle ϕ the dependent variable. However, if present, the alignment effect is so weakly reflected by the available data that the fitting of regression curves has not, to our knowledge, been attempted in the literature. None the less, we show below that non-parametric regression may be used to gain some insight into the question of alignment.

If there is no alignment, then it is expected that the pointing angles should be approximately uniformly distributed over the range $[0°, 90°]$ irrespective of the nearest neighbour distances. This has served as the basis of non-parametric tests used by UMK, CMM1 and CMM2. Both two-sample and one-sample tests have been used: in the former, the data are divided into two groups, one containing those clusters with small nearest neighbour distances $d \leqslant d_0$, with the remainder (with $d > d_0$) in the second group. The distributions of the two associated sets of angles could then be compared using a suitable test statistic; the null hypothesis is that the two sets of angles are similarly distributed, while the alternative is that the distribution of pointing angles associated with small d is more concentrated towards small angles than the angles in the $d > d_0$ group. In the one-sample procedures, only pointing angles from those clusters

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for which $d \leqslant d_0$ are used, and a test for deviations from a uniform distribution is carried out. The most appropriate value of the separation distance d_0 is not known a priori: calculations have therefore typically been reported for a few values in the range $10 \leqslant d_0 \leqslant 40$ Mpc. (We follow CMM1 and CMM2 in assuming $q_0 = 0$ and $H_0 = 100$ km s^{-1} Mpc^{-1} throughout this paper.)

The two sets of authors used different types of distribution-free tests. UMK applied one- and two-sample versions of the Kolmogorov–Smirnov (KS) statistic (see e.g. Conover 1971) to a sample of 46 clusters and found no significant evidence for alignment. The same cluster sample was reconsidered by CMM1, using updated redshift information. The identification of nearest neighbours was revised, and clusters with nearest neighbours farther than the survey region boundary were eliminated; the revised sample consisted of 25 clusters. CMM1 argued that the null hypothesis of a uniform distribution of the pointing angles ϕ is too general, and that the KS tests are therefore not very powerful. They propose instead to use the two-sample Wilcoxon rank sum test or Mann–Whitney (MW) test, for which, in the present context,

$$H_0: \quad P(\phi_L < \phi_H) = 0.5$$
$$H_a: \quad P(\phi_L < \phi_H) > 0.5, \tag{1}$$

where ϕ_L and ϕ_H, respectively, denote pointing angles for clusters with close ($d \leqslant d_0$) and far-off ($d > d_0$) near neighbours. In other words, under the null hypothesis pointing angles from those clusters with $d \leqslant d_0$ are equally likely to be larger or smaller than pointing angles associated with clusters having $d > d_0$. The alternative is an enhanced likelihood for pointing angles from the $d \leqslant d_0$ group to be smaller than angles ϕ associated with $d > d_0$ clusters.

Interestingly, CMM1 found very high MW significance levels for both the original UMK data ($p = 0.004$ for $d_0 = 30$ Mpc) and the revised sample ($p = 0.0004$ for $d_0 = 10$ Mpc and $p = 0.016$ for $d = 20$ Mpc). On the other hand, the KS test results were not significant. (The p-value gives the probability, computed under the assumption that the null hypothesis is true, of the statistic in question having a numerical value in excess of that actually observed.)

A third sample, of size 45, was considered by CMM2.Significance levels of $p = 0.0018$ ($d_0 = 10$ Mpc) and $p = 0.0126$ ($d_0 = 20$ Mpc) were calculated (from non-standard applications of the MW test).

In the present paper, all significance levels for the three data sets are recalculated using standard forms of the test statistics (e.g. Conover 1971). We also calculate the one-sample Wilcoxon statistics, known as the 'Wilcoxon signed rank' or 'Wilcoxon symmetry' (WS) statistics. In the present context, the null hypothesis of the WS test is that the median ϕ associated with $d \leqslant d_0$ is 45°. The alternative is that the median is smaller than 45°.

It should be stressed that the alternative hypotheses in equation (1), and of the WS test, are one-sided (deviation from the null in a particular direction). Similarly, in applying the KS tests, the alternative hypotheses will be that the distribution of ϕ_L is more concentrated towards zero than the distribution of the ϕ_H (two-sample test), or that the distribution of the ϕ_L is more concentrated towards zero than the uniform distribution (one-sample test).

Numerical results are discussed in Section 2, and conclusions are presented in Section 3.

2 DATA ANALYSIS

2.1 UMK data

Fig. 1 shows a scatterplot of pointing angle (vertical axis) against nearest neighbour distance (horisontal axis) for 46 clusters, taken from table 1 of UMK. Superimposed upon this plot is a smooth estimate of the local mean value. The latter was obtained using a local linear method with nearest neighbour bandwidth and tricube weight function (Loader 1999), the bandwidth being chosen by minimizing the prediction sum of squares. If there were indeed an alignment effect, then the expectation is that the local mean value of the ϕ should be smaller than 45° for d close to zero, increase with increasing d and level off to a value near 45° for large d. However, the smooth fit, which is quite variable, provides no visual evidence in support of alignment.

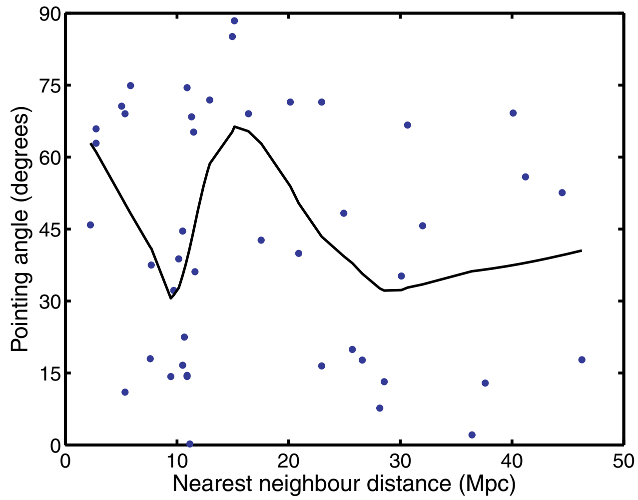

Figure 1. The pointing angles associated with nearest neighbour distances for 46 clusters of galaxies. The data were taken from table 1 of UMK. A non-parametric estimate of the local mean pointing angle is also shown.

Fig. 2 is a plot of the empirical distribution function (EDF) of pointing angles at $d \leqslant 15$ Mpc together with a plot of the distribution function of the uniform distribution function. There is no visual evidence of any substantial systematic difference. This is confirmed by the application of the WS test, which gives p-values of 0.465 and 0.321 at $d_0 = 10$ and 15 Mpc, respectively. Since the hypothesis of symmetry around 45° cannot be rejected, there is again no support for alignment.

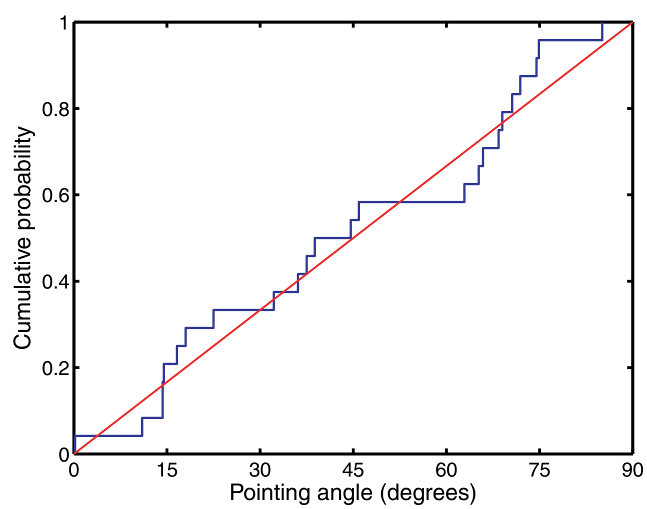

Figure 2. The empirical distribution function of pointing angles in Fig. 1 for those clusters with nearest neighbours within 15 Mpc. The cumulative distribution function for a uniform distribution (straight line) is also shown.

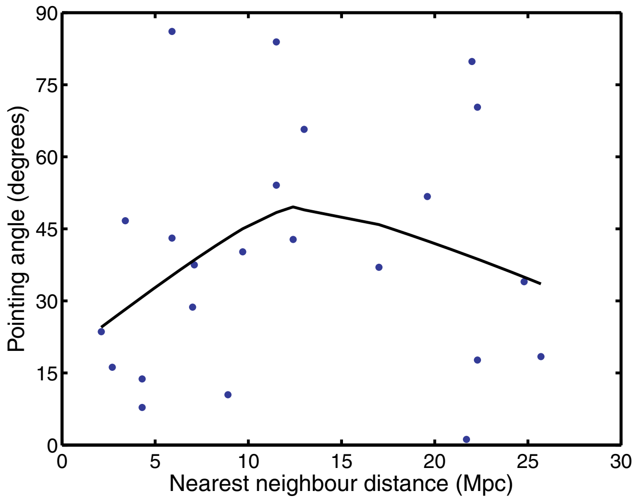

Figure 3. The pointing angles associated with nearest neighbour distances for 25 clusters of galaxies. The data were taken from table 1 of CMM1. A non-parametric estimate of the local mean pointing angle is also shown.

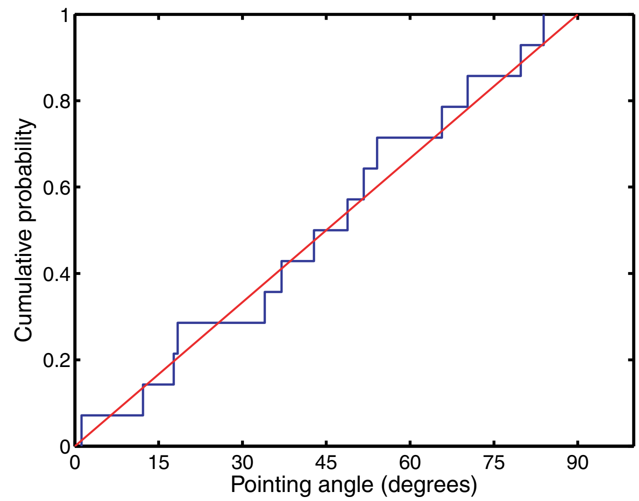

Figure 5. The empirical distribution function of pointing angles in Fig. 3, for those clusters with nearest neighbours farther than 10 Mpc. The cumulative distribution function for a uniform distribution (straight line) is also shown.

A glance at fig. 1 in UMK shows that the optimal contrast between the ϕ_L and ϕ_H collections of pointing angles is obtained with d_0 slightly less than 15, say $d_0 = 14$ Mpc. (This is confirmed by a more formal analysis.) Comparison of the ϕ_L and ϕ_H samples using the MW statistic gives a significance level of 0.45.

2.2 CMM1 data

Next, we examine the sample of 25 clusters in table 1 of CMM1. Fig. 3 shows a scatterplot of pointing angle against nearest neighbour distance together with a local smooth. There is some evidence of alignment here because the local mean increases up to a distance of $d = 13$ Mpc. Further support for alignment comes from Fig. 4, which shows the EDF of pointing angles for $d_0 = 10$ Mpc together with the uniform distribution function over the interval [0°, 90°]. Note that the former lies well above the latter over almost the full range. This fact suggests that these pointing angles come from a distribution that is more concentrated than uniform and that has a mean of somewhat less than 45°. The KS one-sample test produces a p-value of 0.026, which provides reasonably strong evidence in support of the preceding conjectures. Considering only pointing angles with $d > 10$ Mpc, there is no evidence to refute the conjecture that these are uniformly distributed (see Fig. 5). Overall, we conclude that there is moderately strong evidence of alignment for clusters with nearest neighbour distances $d \leqslant 10$ Mpc but none at distances $d > 10$ Mpc. These conclusions are also supported by an application of the WS test. The p-values corresponding to the subsamples of pointing angles at $d_0 = 10$, 20 and 30 Mpc are 0.032, 0.180 and 0.124, respectively.

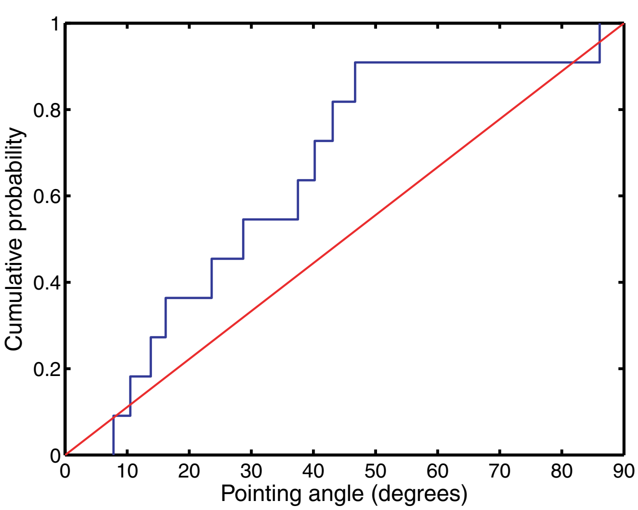

Figure 4. The empirical distribution function of pointing angles in Fig. 3 for those clusters with nearest neighbours within 10 Mpc. The cumulative distribution function for a uniform distribution (straight line) is also shown.

By contrast, our MW comparisons detect no significant differences between the mean of the respective sets of pointing angles. The two-sample KS test detects the difference (see Figs 4 and 5) between the $d \leqslant 10$ and $d > 10$ sets of pointing angles at a marginal level of $p \approx 0.1$. The standard MW test significance level is virtually the same.

2.3 CMM2 data

Finally, we examine the 45-cluster data set constructed in CMM2. First, note that an amendment to the sample selected from table 2 in CMM2 is required: since only clusters with well-determined pointing angles are selected for analysis, cluster Abell 2151 should be included while cluster Abell 1983 should be excluded. Fig. 6 shows a scatterplot of pointing angle against nearest neighbour distance together with a local smooth. The rise between $d = 0$ and 15 followed by a gradual flattening out is again suggestive of alignment. Fig. 7 shows the same scatterplot with a vertical line drawn at $d = 11$. The latter value was obtained as the maximizer of the two-sample KS statistic (comparison of ϕ_L and ϕ_H samples over all separation distances d_0). The corresponding test statistic, namely the maximum KS statistic over all separation distances, produces a significance level of 0.012. This statistic takes into account the multiple tests performed over the various values of d_0.

Note the absence of data in the upper left hand and lower right hand corners of Fig. 7. In the analysis that follows, we will refer to the 23 ϕ_L values at $d \leqslant 11$ Mpc as 'block 1' and to the 22ϕ_H values at $d > 11$ Mpc as 'block 2'. The maximum ϕ-value in block 1 is

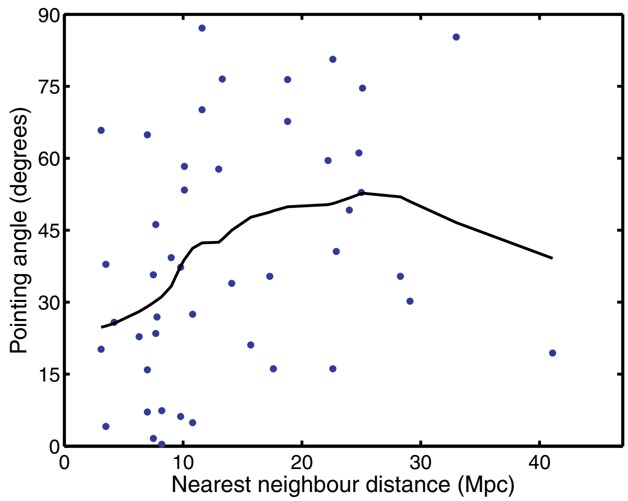

Figure 6. The pointing angles associated with nearest neighbour distances for 45 clusters of galaxies. The data were taken from table 2 of CMM2. A non-parametric estimate of the local mean pointing angle is also shown.

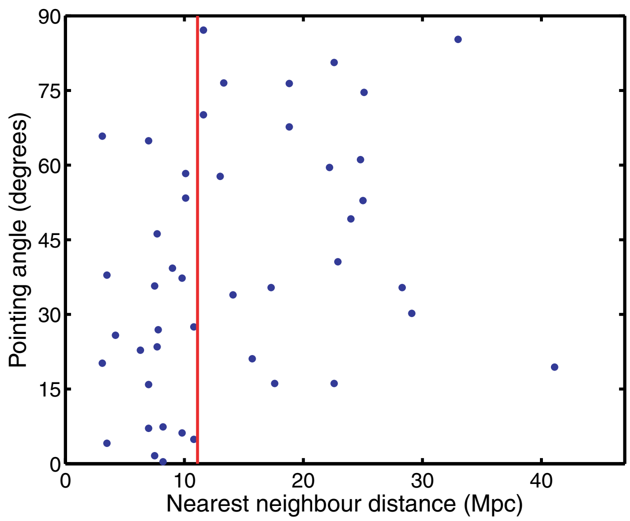

Figure 7. The data in Fig. 6, but with the separating line $d_0 = 11$ Mpc which maximizes the difference between the empirical distribution functions of pointing angles with $d \leqslant d_0$ and $d > d_0$.

65°.8, and there is a chance of only $(66/90)^{23} = 8 \times 10^{-4}$ that 23 values from a uniform distribution over $[0°, 90°]$ will all be less than 66°. Similarly, the minimum ϕ-value in block 2 is 16°.1, and there is a chance of only $(1 - 16/90)^{22} = 0.014$ that 22 values from a uniform distribution over $[0°, 90°]$ will all be larger than 16°. Thus, it seems safe to conclude that the absence of data in the upper left hand and lower right hand corners is 'real' and not merely due to sampling fluctuations.

Fig. 8 shows the EDFs of the data in blocks 1 and 2 together with the uniform distribution functions over the intervals $[0°, 66°]$ and $[16°, 90°]$, respectively. In neither case is there a statistically significant deviation from uniformity, and it is therefore reasonable to suggest that the angles in blocks 1 and 2 come from uniform distributions over $[0°, 66°]$ and $[16°, 90°]$, respectively. Thus, the evidence points to alignment up to distances $d \leqslant 11$ but not beyond that. None the less, the lack of small ϕ values for larger nearest neighbour distances $d > 11$ is not exactly in accord with expectation.

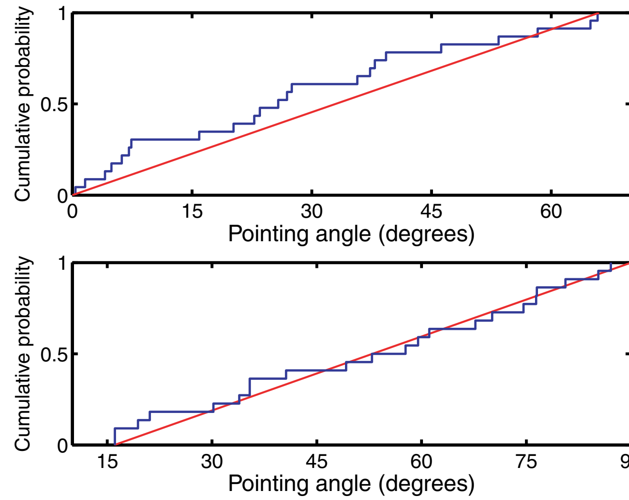

Figure 8. Top panel: the empirical distribution function of the pointing angles in Fig. 7, for those clusters with nearest neighbours within 11 Mpc (left of the vertical line in Fig. 7). The cumulative distribution function for a uniform distribution over $[0°, 66°]$ (straight line) is also shown. Bottom panel: the EDF of the pointing angles in Fig. 7, for those clusters with nearest neighbours farther than 11 Mpc (right of the vertical line in Fig. 7). The cumulative distribution function for a uniform distribution over $[16°, 90°]$ (straight line) is also shown.

Table 1. Significance levels of the WS statistic applied to various partitions of the CMM2 data set.

	p-values	
d_0 (Mpc)	$d \leqslant d_0$	$d > d_0$
10	0.001	0.836
20	0.027	0.784
30	0.066	0.251

The preceding conclusions are also confirmed by application of the WS test. The p-values derived for three choices of d_0 are given in Table 1. For the sake of interest, the significance levels of WS tests applied to the pointing angles associated with $d > d_0$ are also shown.

We observe that the p-values corresponding to large nearest neighbour distances are all in excess of 0.25, which suggests that the distributions of pointing angles associated with $d > 10$ Mpc are consistent with symmetry around 45°. Thus, there is no evidence of alignment much beyond $d = 10$ Mpc. Of course, the small p-values corresponding to $d \leqslant 20$ and 30 Mpc are merely due to the carry-over effect of the highly asymmetric distribution of pointing angles at $d \leqslant 10$ Mpc. If such a highly asymmetric distribution is mixed with a uniform distribution, the result is still an asymmetric distribution. Thus, it would be incorrect to conclude that alignment exists up to $d \leqslant 30$ Mpc. The proper manner in which to determine the range of alignment is to compare the distributions of the pointing angles of clusters at distances $d \leqslant d_0$ with those of clusters at distances $d > d_0$.

3 DISCUSSION

The p-values calculated for the various tests and data sets are summarized in Table 2. For interest, the p-values of one-sample tests

Table 2. A summary of the significance levels of the various tests applied to the three published data sets. For the UMK data, $d_0 = 14$ Mpc; for the CMM1 and CMM2 data, $d_0 = 10$ Mpc. The designations 'KS1' and 'KS2' indicate the KS one- and two-sample tests, respectively; 'WS' and 'MW' are the Wilcoxon symmetry and the Mann–Whitney tests, respectively. In the case of the one-sample tests, results for pointing angles associated with $d > d_0$ are given in brackets.

Data set	Test	p
UMK	KS1	0.253 (0.387)
	WS	0.206 (0.296)
	KS2	0.732
	MW	0.446
CMM1	KS1	0.026 (0.679)
	WS	0.032 (0.438)
	KS2	0.096
	MW	0.104
CMM2	KS1	0.001 (0.946)
	WS	0.001 (0.836)
	KS2	0.005
	MW	0.001

for clusters with $d > d_0$ are also given – it is no great surprise that these are not significant (although see below).

It is interesting that the one-sample (KS1, WS) test results are generally more significant than the two-sample results (KS2, MW). This can be ascribed to the fact that the former are more specific – the null hypotheses are uniformity and a median of 45°, respectively. By contrast, the two-sample procedures test for completely unspecified differences between two data sets, and hence may be less powerful in specific cases. It is also noteworthy that the significance levels of the one-sample tests are comparable, as are the p-levels of the two-sample procedures.

We conclude with a few remarks. First, fig. 1 in UMK shows an updated version of optical $d - \phi$ data from Bingelli (1982). Comparison with the similar fig. 2 of CMM1 and fig. 1 of CMM2 shows a more pronounced deficiency of observations with small d and large ϕ in the Bingelli (1982) data. In particular, for $d \leqslant 15$ only one pointing angle was measured to be larger than 45°. Given the results above, it seems likely that tests for alignment should give significant results. Unfortunately, the updated Bingelli (1982) data used by UMK appear to be only available in the form of figures.

Second, it is worth reiterating that rejection of the null hypothesis of 'no alignment' at some large value of d_0 does not necessarily imply that the alignment extends to that particular d_0. A strong effect at small d_0 will give rise to test statistics which are still significant at larger d_0, albeit at a reduced level due to attenuation by random pointing angles.

Third, none of the statistical tests incorporates the effects of measurement errors. These are present in both ϕ (see particularly CMM2, table 2, where determinations by different authors are compared) and d (where both misidentification of nearest neighbours and calculation of d may play a role).

Fourth, it was shown in Section 2.3 that a very specific test rejects the null hypothesis of a uniform distribution of the pointing angles ϕ_H of clusters with $d_0 > 11$ Mpc. The implication is that there are too few small pointing angles at large nearest neighbour distances, which appears unlikely to be a real physical effect. Taken in isolation, this could have cast doubt on the significant results for $d < d_0$; however, the significance levels attained by tests of the CMM2 data, in particular at small d, are resounding.

Fifth, the significance levels in Tables 1 and 2 apply to pre-selected values of d_0. The significance levels associated with d_0 chosen to maximize any of the statistics (KS1, KS2, MW or WS) are considerably lower. For example, as seen above, the p-value of the KS2 statistic maximized over d_0 for the CMM2 data is 0.012, whereas the significance level is 0.005 if $d_0 = 10$ Mpc is specified without prior testing.

Finally, the details of the samples selected by various authors have influenced the results obtained. This problem will be alleviated if the reliability of each observation can be assessed, and this information incorporated into the analysis. Increased sample sizes would also help.

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