Hardie, K.A. & Witbooi, P.J. (2013). The fibre of a pinch map in a model category.
Applied Categorical Structures, 21: 867 – 875.
http://dx.doi.org/10.1007/s10485-012-9276-5

UNIVERSITY of the WESTERN CAPE

The fibre of a pinch map in a model category

Keith A. Hardie and Peter Joseph Witbooi

Abstract

In the category of pointed topological spaces, let F be the homotopy fibre of the pinching map $X \cup CA \to X \cup CA/X$ from the mapping cone on a cofibration $A \to X$ onto the suspension of A. Gray (Proc Lond Math Soc (3) 26:497–520, 1973) proved that F is weakly homotopy equivalent to the reduced product $(X, A)_\infty$. In this paper we prove an analogue of this phenomenon in a model category, under suitable conditions including a cube axiom.

1 Introduction

In the paper [5] we have defined *reduced powers* X_n of an object X in a model category **C** and, assuming a certain cube axiom holds in **C**, established the weak equivalence,

$$X_\infty \xrightarrow{\sim} \Omega\Sigma X\,, \qquad (1.1)$$

so generalising an influential result on reduced product spaces due to James [7]. See also the generalizations in [3]. Although the argument began by constructing an analog of (X, A) (i.e. of Gray's relative version of the James construction on a cofibration $A \to X$ [4]). We were not able in [5] to recover in **C** a weak equivalence

$$(X, A)_\infty \xrightarrow{\sim} F\,, \qquad (1.2)$$

where F is the homotopy fibre of the pinching map $X \cup CA \to :E A$. The situation is remedied here by showing that the desired result indeed holds under a mild additional assumption.

2 The Cube Axiom

Quillen [8] described an abstract approach to homotopy theory enabling analogous theories to be defined in categories other than the category of topological spaces and continuous maps. A *model category* consists of a category **C** with all small limits and colimits together with three distinguished classes of morphisms, *we*, *cof*, *f ib*, called

References

1. Baues, H.J.: Algebraic Homotopy. Cambridge University Press, Cambridge, UK (1989)
2. Bourn, D., Janelidze, G.: Protomodularity, descent, and semidirect products. Theory Appl. Categ. **4**(2), 37–46 (1998, electronic)
3. Fantham, P., James, I., Mather, M.: On the reduced product construction. Canad. Math. Bull. **39**(4), 385–389 (1996)
4. Gray, B.: On the homotopy groups of mapping cones. Proc. Lond. Math. Soc. (3) **26**, 497–520 (1973)
5. Hardie, K.A., Witbooi, P.J.: Reduced product objects in a model category. Bull. Belg. Math. Soc. Simon Stevin **12**(1), 141–152 (2005)
6. Hovey, M.: Model Categories. Mathematical Surveys and Monographs, vol. 63. Amer. Math. Soc. (1999)
7. James, I.M.: Reduced product spaces. Ann. Math. **62**(2), 170–197 (1955)
8. Quillen, D.G.: Homotopical Algebra. Lecture Notes in Mathematics, vol. 43. Springer, New York (1967)