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What can be deduced from open cluster metallicity measurements?

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ABSTRACT

The dependence of [Fe/H] on galactocentric distance, distance from the galactic mid-plane and age is studied. Both ordinary least-squares and non-parametric regression in the form of a 'generalized additive model' are used. The radial metallicity slope is found to be shallower than previously claimed in the literature, and there is a significant abundance gradient perpendicular to the galactic plane. There may be a tendency for metallicity to increase with cluster age.

Key words: methods: statistical – Galaxy: abundances – open clusters and associations: general.

1 INTRODUCTION

A number of authors have used measurements of [Fe/H] of open clusters to infer the dependence of the galactic disc metallicity on age (e.g. Carraro, Ng & Portinari 1998), galactocentric distance and distance above the galactic plane (e.g. Cheng, Hou & Wang 2003). The aim of this paper is a critical assessment of what can be learnt about these issues from the currently available data.

It is, of course, important to see the open cluster abundance research in the broader context of other galactic metallicity studies. In particular, many other tracers of the radial galactic disc metallicity gradient have also been used: amongst these are Cepheid pulsators (Andrievsky et al. 2004), planetary nebulae (PNe; Perinotto & Morbidelli 2006), OB stars (Daflon & Cunha 2004), red giants (Carney et al. 2005), H II regions (Vílchez & Esteban 1996; Henry & Worthey 1999), G and K giants (Neese & Yoss 1988) and supernova remnants (Fesen, Blair & Kirshner 1985). (Further relevant references can be found in these papers.) Generally a small negative slope is found, although it is of marginal significance for some types of object (e.g. the PNe studied by Perinotto & Morbidelli 2006). A point of particular interest is the possibility that the radial dependence of the metallicity can be better described by a step function, discontinuous near $R_G \sim 10$ kpc, rather than the commonly assumed linear form (Twarog, Ashman & Anthony-Twarog 1997; Corder & Twarog 2001; Andrievsky et al. 2004). It has also been suggested that the abundance slope is close to zero for larger galactocentric distances (e.g. Henry & Worthey 1999; Yong, Carney & de Almeida 2005) although it may be very steep for small R_G (e.g. Vílchez & Esteban 1996).

The literature on metallicity gradients in the direction perpendicular to the galactic disc is also quite extensive. Very brief summaries can be found in Rana (1991) and Henry & Worthey (1999). A more extensive discussion in Du et al. (2004), based on photometry of F/G stars, makes it clear that the gradient is probably a function of height above the plane, increasing from ~ -0.4 dex kpc^{-1} near the plane to ~ -0.1 dex kpc^{-1} or shallower for $z > 5$ kpc. Results for open clusters have been, and continue to be, disparate: Cheng et al. (2003) derived a slope of -0.30 ± 0.05 dex kpc^{-1}, while Salaris, Weiss & Percival (2004) find no correlation between [Fe/H] and distance z from the mid-plane.

A discussion of the relationship between the age and metallicity of open clusters can be found in Yong et al. (2005), which also contains reference to earlier papers. The conclusion is that there is no obvious dependence of [Fe/H] on age. A similar result was obtained from observations of a very large number of field stars summarized by Andersen, Nordström & Mayor (2005); the authors point out how previous contrary conclusions resulted from biased samples. The statement by Carraro et al. (1998) that there is an '...upturn of the metallicity of the open clusters, possibly with a peak near $t \approx 8$ Gyr ...' is particularly intriguing, and we will return to it.

Carraro et al. (1998) also consider interaction between age and radial metallicity gradients of open clusters – i.e. the possibility that the radial gradient may be a function of time – and conclude that the gradient appears not to have changed much over time.

Obvious advantages of using data for star clusters are (at least in principle) the more accurately determined distances, ages and metallicities. Furthermore, clusters are less susceptible to orbital diffusion than field stars, hence there ought to be less confusion between the effects of age and metallicity gradients. Lastly, clusters span a very wide range of galactocentric distances.

The data analysed in this paper are taken from the latest version (2.7; 2006 October) of the 'New Catalog of Optically Visible Open Clusters and Candidates' (see Dias et al. 2002). Abundances are currently available for 147 clusters; for all but two of these there are also age estimates. Heliocentric distance d and position (galactic longitude ℓ and latitude b) are then used to calculate galactocentric distance R and distance z from the galactic mid-plane:

$$R = [\mathrm{R}_0^2 + (d\cos b)^2 - 2\mathrm{R}_0 d\cos b\cos\ell]^{1/2},$$

$$z = d|\sin b|.$$

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The distance of the Sun from the galactic centre is assumed to be $R_0 = 8.5$ kpc. The age variable A to be used is

$$A = \log_{10}(\text{age}).$$

The next two sections of the paper, respectively, deal with the results of traditional linear regression and non-parametric regression of [Fe/H] on R, z and A. Concluding remarks are made in Section 4.

2 ORDINARY LEAST-SQUARES REGRESSION

Let y be the dependent variable ([Fe/H] in the present context) and X_j ($j = 1, 2, \ldots, K$) K independent variables (combinations of R, z and A). The models discussed in this section are of the well-known linear regression form

$$y = \alpha + \sum_j \beta_j X_j + \text{error}, \qquad (1)$$

where α and the β_j are constants. The ith data point is denoted by $(X_{1i}, X_{2i}, \ldots, X_{Ki}; y_i)$ for $i = 1, 2, \ldots, N$.

Plots of metallicity against galactocentric distance, distance from the plane and cluster age can be seen in Figs 1–3. Although useful – particularly as regards to the identification of outlying data – these cannot be taken as fully illustrative of the dependence of [Fe/H] on R, z or A, due to the interdependence of the three independent variables. Correlation coefficients (Table 1) are all very highly significant ($p \leqslant 0.004$). Interpretation of the correlations of R, z and A with the metallicity is bedevilled by the interactions between the three independent variables. A useful adjunct is provided by the *partial* correlations with the metallicity. The partial correlations between X_j and y is the direct correlation between the two variables, i.e. the correlation left after discounting the influence of all the other independent variables. The partial correlations are listed in Table 2.

Given the p value of 0.98 associated with A in Table 2, it is not surprising that the coefficient of A is non-significant in a linear regression of [Fe/H] on R, z and A. The result is

$$[\text{Fe/H}] = 0.20(0.081) - 0.026(0.009)R - 0.12(0.057)z$$
$$\sigma = 0.18, \qquad (2)$$

where standard errors of estimated coefficients are given in brackets.

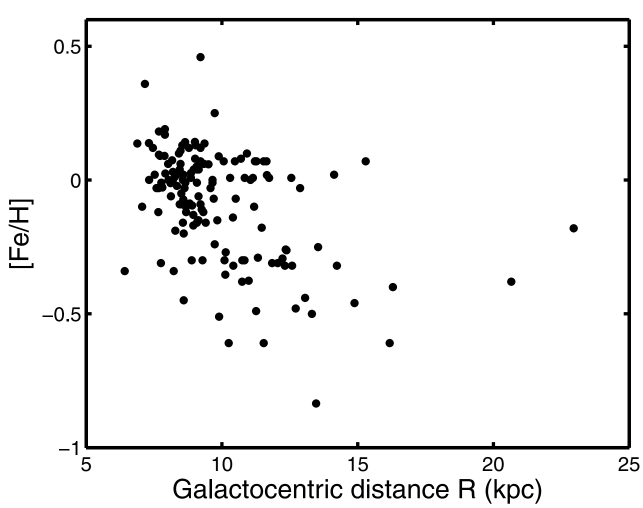

Figure 1. Metallicity as a function of galactocentric distance for the 147 clusters with abundance determinations.

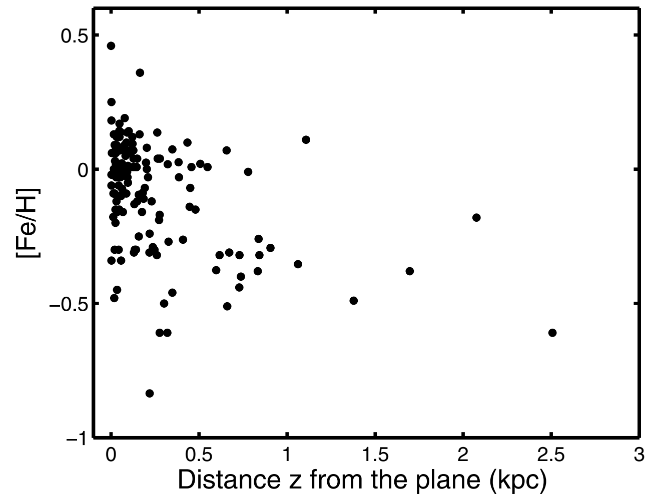

Figure 2. Metallicity as a function of the distance from the galactic mid-plane.

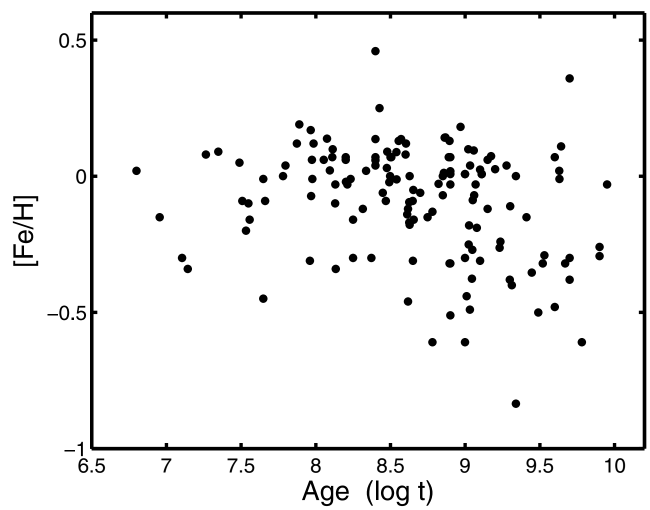

Figure 3. Metallicity as a function of the cluster age.

Table 1. The simple correlations between the four variables of interest – galactocentric distance R, distance z from the galactic plane, log (age) of the cluster and metallicity [Fe/H].

	R	z	A	[Fe/H]
R	1.00	0.70	0.44	−0.45
z		1.00	0.49	−0.42
A			1.00	−0.24

It is noteworthy that the coefficient of R, despite being highly significant, is much closer to zero than the values −0.063(0.008) and −0.055(0.019) found by Cheng et al. (2003) and Salaris et al. (2004), respectively. One reason for the difference is that those authors may not have allowed for the R–z correlation, which requires simultaneously regressing [Fe/H] on R and z. However, using the present data set, if we regress [Fe/H] on R only, the coefficient is −0.039(0.006) which is still very significantly different from the two earlier determinations: clearly, the exact data sets used also play a major role. This is easily verified by leaving out the most

Table 2. As for Table 1, but showing the partial correlations of the independent variables with the metallicity. The last column contains the significance levels.

	[Fe/H]	p
R	−0.24	0.004
z	−0.16	0.06
A	−0.002	0.98

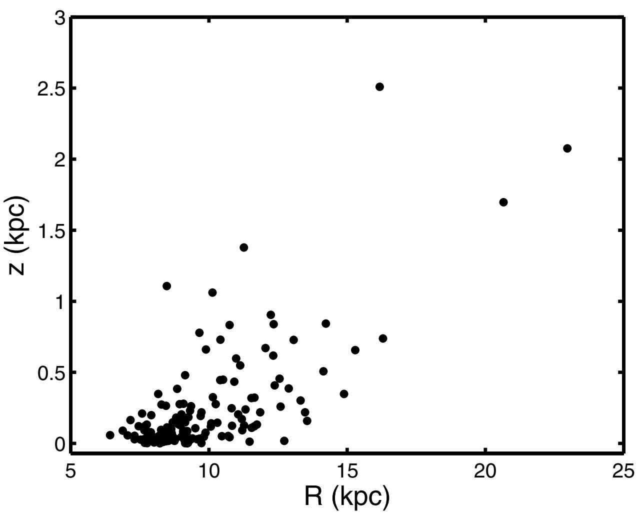

Figure 4. The positions of the complete sample of 147 clusters in the R–z plane. The reader's attention is drawn to the three outlying points at large R and large z.

extreme R value (see Fig. 1): the metallicity–distance slope changes to -0.048 ± 0.007. The point is further underscored by leaving out the cluster furthest from the galactic mid-plane: (2) changes to

$$[\mathrm{Fe/H}] = 0.21(0.081) - 0.027(0.009)R - 0.10(0.066)z$$
$$\sigma = 0.18,$$

i.e. the coefficient of z is no longer significant.

Inspection of Fig. 4 shows clearly that that there are three high-leverage points in R–z space: clusters at large galactocentric distances, which also lie at considerable ($z > 1.5$ kpc) height above the plane. Deleting these three observations gives

$$[\mathrm{Fe/H}] = 0.32(0.085) - 0.038(0.009)R - 0.17(0.067)z$$
$$\sigma = 0.18 \qquad (3)$$

which is not strongly dependent on the presence of any one data point.

In essence the extreme region in R–z space is too thinly populated to obtain reliable estimates over the entire region covered by the observations. In order to guarantee robust results which are not unduly influenced by a few outlying observations the restrictions $R < 17$ kpc and $z < 1.5$ kpc are imposed hereafter. This leaves 144 clusters, 142 of which also have age estimates.

Regressing the metallicity only on R for the reduced data set gives a slope of -0.050 ± 0.008, about 30 per cent steeper than that found in (3), emphasizing the importance of including z in the regression. The dependence on z in (3) is significant at the 1 per cent level (contrasting with the result obtained by Salaris et al. 2004), but considerably shallower than the -0.295 ± 0.050 found by Cheng et al. (2003). It is not clear whether the latter authors allowed for the simultaneous dependence on R, as is necessary.

There are two statistics associated with the regression models which are worth mentioning, particularly for the purpose of comparison with the results in Section 3. The first is the 'adjusted coefficient of determination':

$$R_{\mathrm{a}}^2 = 1 - \frac{\sigma_{\mathrm{m}}^2}{\sigma_0^2},$$

where

$$\sigma_0^2 = \frac{1}{N-1}\sum_{k=1}^{N}(y_k - \overline{y})^2$$

and

$$\sigma_{\mathrm{m}}^2 = \frac{1}{N-p-1}\sum_{k=1}^{N}\left(y_k - \widehat{\alpha} - \sum_{j=1}^{p}\widehat{\beta}_j X_{jk}\right)^2$$

are, respectively, the variance estimated without and with the model fitted to the N data points. The statistic R_{a}^2 therefore measures the proportion of the variation in the data explained by the model – see e.g. Montgomery, Peck & Vining (2001). Examination of the formulae shows that $R_{\mathrm{a}}^2 = 0$ for completely uninformative models ($\sigma_{\mathrm{m}}^2 = \sigma_0^2$) and $R_{\mathrm{a}}^2 = 1$ for a perfect model ($\sigma_{\mathrm{m}}^2 = 0$). Intermediate models obviously have $0 < R_{\mathrm{a}}^2 < 1$.

The second quantity of interest is the 'Akaike information criterion' (AIC; e.g. Burnham & Anderson 2002; Wood 2006). A typical form is

$$\mathrm{AIC} = N \log \sigma_{\mathrm{m}}^2 + 2(K+1);$$

the first term measures how well the model fits, and the second the number of model parameters required to accomplish the fit. Small values of K mean less complex models, and small values of σ_{m}^2 mean small residual variance, hence the best models are those giving small AIC.

For the full model (i.e. including R, z and A in 1) $R_{\mathrm{a}}^2 = 0.235$, while exclusion of the non-significant term in age gives a minimally smaller $R_{\mathrm{a}}^2 = 0.234$. Given the small differences in explanatory power, it is to be expected that the AIC is smaller for the less complex model containing only R and z (−83.0 versus −82.2 for the full model). Retaining only R gives AIC = −78.70, which is substantially inferior.

Two points are worth making: first, the values of R_{a}^2 are quite low, meaning that although the statistical models are highly meaningful, they only describe a relatively small part (about a quarter) of the variance in the data. Second, in the present context the value of the AIC is not meaningful on any absolute scale – rather, relative values are used for intercomparison of models.

We turn to a brief examination of the dependence of metallicity on cluster age. The relation between galactocentric distance and A is illustrated in Fig. 5. Examination of the diagram shows that for distances $R < 9.5$ kpc or so there is a wide range of cluster ages, and that these seem independent of R. The latter impression is borne out by the fact that for these 85 clusters the correlation between distance and age is −0.006. The partial correlations with metallicity are

	[Fe/H]	p
R	−0.07	0.95
z	−0.10	0.38
A	0.28	0.01

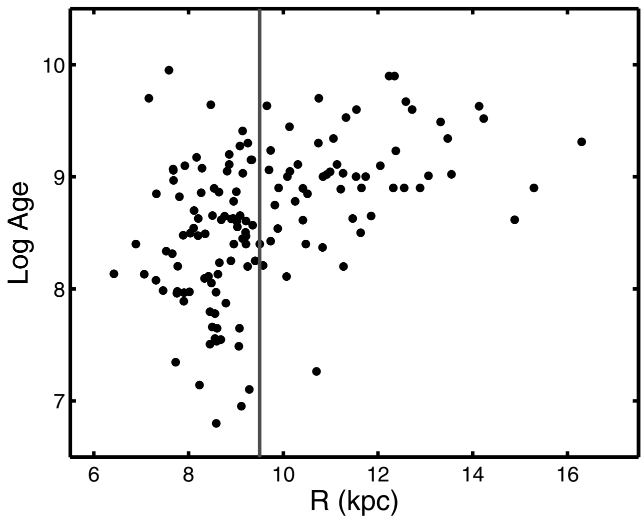

Figure 5. Cluster age plotted against galactocentric distance. Ages to the left of the vertical line at $R = 9.5$ kpc are uncorrelated with distance.

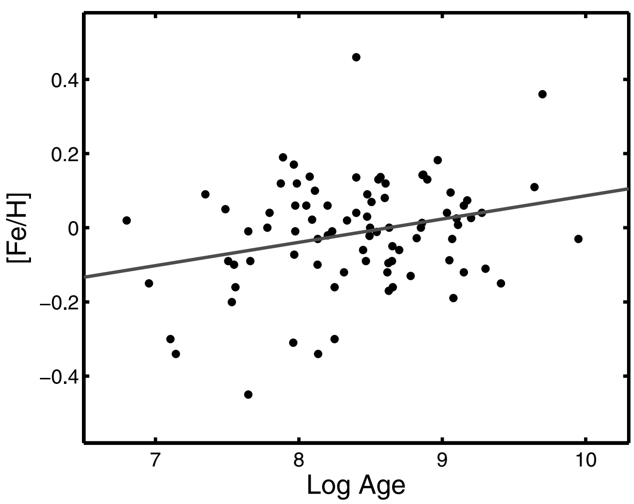

Figure 6. The relation between metallicity and cluster age for the subsample of 85 clusters with $R < 8.5$ kpc.

The independence of [Fe/H] and R for $R < 9.5$ can be verified in Fig. 1, while Fig. 6 shows the relation between age and metallicity for the same subset of the data. Not surprisingly, regressing [Fe/H] on R, z and A gives p values for the first two coefficients of 0.52 and 0.38, respectively, while A is a highly significant regressor with $p = 0.01$. Excluding the two non-significant variables the result

$$[\mathrm{Fe/H}] = -0.54(0.20) + 0.06(0.024)A \quad \sigma = 0.14 \tag{4}$$

is obtained. This implies an *increase* in metallicity with age, recalling the remark by Carraro et al. (1998) quoted in the Introduction. Although suggestive, the result should be treated with some caution as there are obviously many ways to subdivide the data, with different partitions giving different regression results.

3 NON-PARAMETRIC REGRESSION

A very powerful generalization of (1) is the 'Generalized Additive Model' (GAM):

$$y = \alpha + \sum_{j=1}^{p} f_j(X_j) + \text{error}, \tag{5}$$

where the forms of the functions f_j are determined by the data – see e.g. Hastie & Tibshirani (1990) and Wood (2006). The f_j are, in general, non-linear. We follow the latter, since there is very convenient-to-use R statistical software available (package MGCV).

A key element of fitting GAMs is determination of the forms of the unspecified functions f_j. The f_j are estimated by fitting the data with functions from some convenient family, such as splines, or local low-order polynomials. In the Wood (2006) implementation the basis functions are 'thin plate regression splines' (TPRSs). The 'backfitting algorithm' is an iterative scheme for fitting GAMs.

(i) Estimate α by $\overline{y}$ and set $f_j \equiv 0$ for all j.
(ii) Estimate $f_1^{(1)}$ by fitting to $y - \overline{y}$.
(iii) Estimate $f_2^{(1)}$ by fitting to $y - \overline{y} - f_1(X_1)$.
(iv) Continue in this fashion, estimating $f_k^{(1)}$ by fitting to $y - \overline{y} - \sum_{i=1}^{k-1} f_i(X_i)$, until a full set of first estimates $f_1^{(1)}, f_2^{(1)}, \ldots, f_p^{(1)}$ have been obtained.
(v) An improved set of estimates follow by fitting $f_j^{(2)}$ $(j = 1, 2, \ldots, p)$ to

$$y - \overline{y} - \sum_{i=1}^{j-1} f_i^{(2)}(X_i) - \sum_{i=j+1}^{p} f_i^{(1)}(X_i).$$

(vi) Repeat step (v) to obtain successive improved sets of estimates $\{f_1^{(3)}, f_2^{(3)}, \ldots, f_p^{(3)}\}$; $\{f_1^{(4)}, f_2^{(4)}, \ldots, f_p^{(4)}\}, \ldots$
(vii) The procedure is terminated when convergence is achieved, i.e. when further repetition of step (v) gives minimal or no change in the estimated f_j.

Inspection of the AICs of the various possible models shows the two best are

$$y = \alpha + f_R(R) + f_z(z) + f_A(A) + \text{error}$$
$$\text{AIC} = -91.8 \quad R_\mathrm{a}^2 = 0.34 \tag{6}$$

and

$$y = \alpha + g_R(R) + g_z(z) + \text{error} \quad \text{AIC} = -86.1 \quad R_\mathrm{a}^2 = 0.28, \tag{7}$$

where f and g are the respective non-parametric functions for the two models. The Akaike criteria for other combinations of f_R, f_z and f_A lie in the interval [−78.7, −74.6], except for the model containing only f_A, which has AIC = −55.5. The models (6) and (7) are therefore best by some margin. However, despite their superiority also to the models of Section 2 (according to the AIC) the percentage variation described is still quite low ($R_\mathrm{a}^2 \leqslant 0.34$). The three non-parametric functions in (6) are plotted in Figs 7–9, together with their estimated ±2 standard error bounds. The estimated dependence on logarithmic age is perfectly linear, while the other two functions are non-monotonic. It is noteworthy that the errors on f_A are in fact consistent with a function which is zero for all A.

An oddity of the model is that the formal significance of the term in A is only 9.5 per cent – but deleting it leads to the model (7) which is substantially inferior. For the sake of interest Fig. 10 shows the estimate of $g_R(R)$ for the model (7). The function is linear, with slope -0.035 ± 0.010, i.e. very similar to that in (3). The form of g_z in (7) is virtually indistinguishable from the function plotted in Fig. 8.

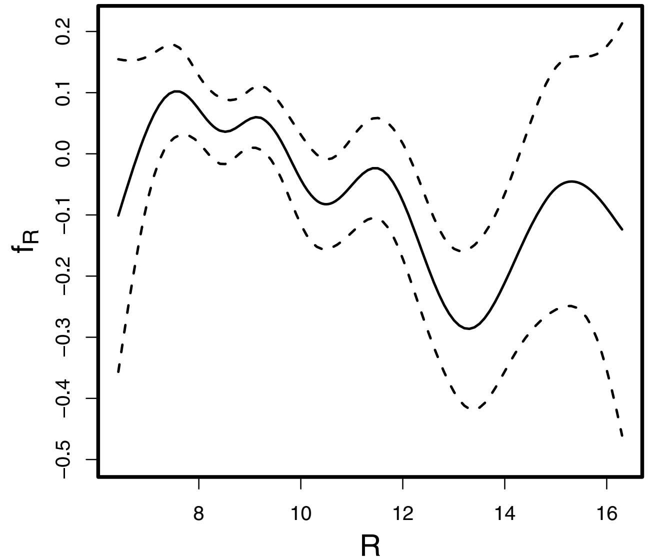

Figure 7. The non-parametric regression function $f_R(R)$ in (6). The dashed lines are ± 2 standard error bounds.

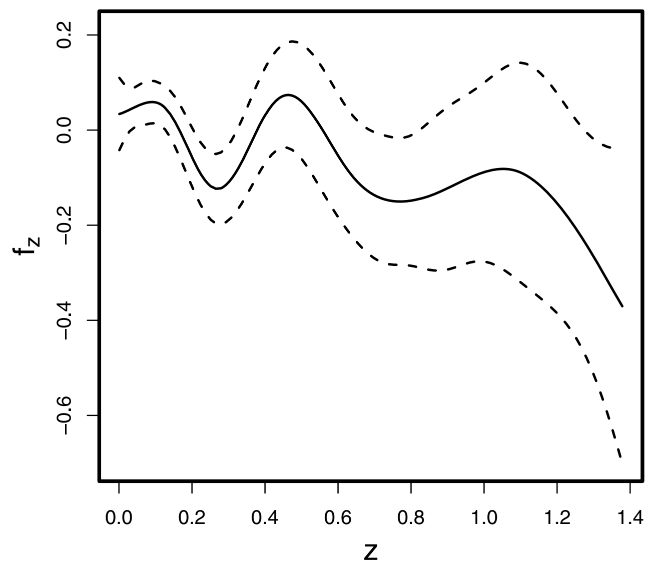

Figure 8. The non-parametric regression function $f_z(z)$ in (6). The dashed lines are ± 2 standard error bounds.

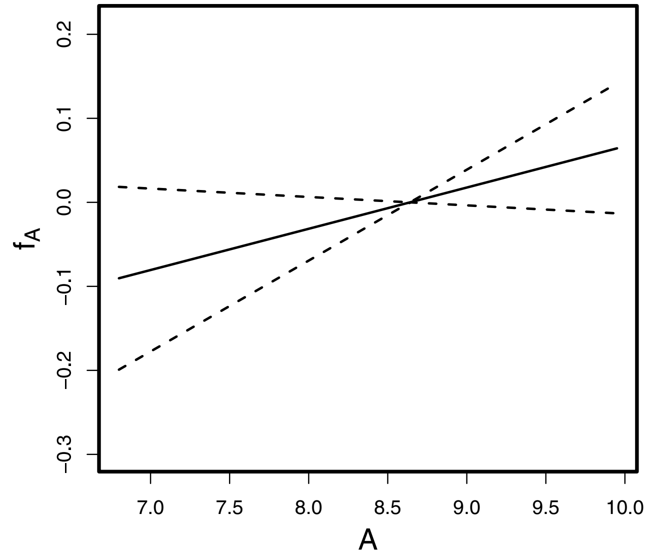

Figure 9. The non-parametric regression function $f_A(A)$ in (6). The dashed lines are ± 2 standard error bounds.

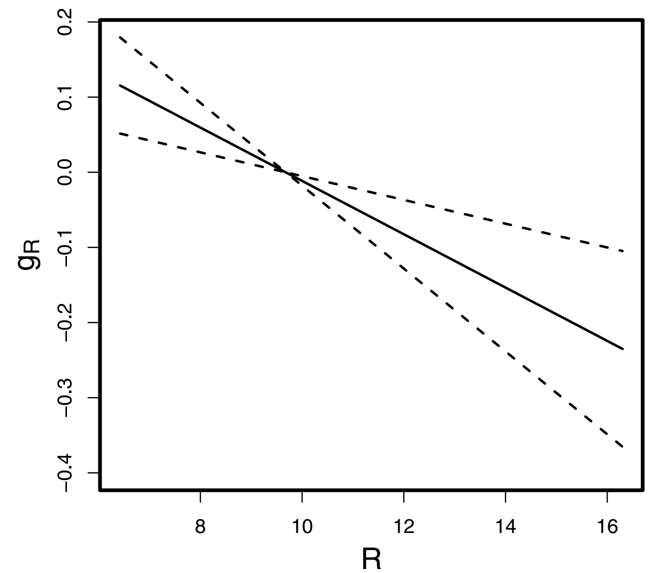

Figure 10. The non-parametric regression function $g_R(R)$ in (7). The dashed lines are ± 2 standard error bounds.

4 CONCLUSIONS

In a nutshell, in the authors' view the available open cluster data are not yet sufficient to fit definitive models of the spatial and/or age dependence of the galactic metallicity. It is conceivable that this goal may be permanently unattainable, due to the scale of random variability in metallicities.

Nonetheless, a few cautious conclusions may be drawn.

(1) Both parametric and non-parametric regression models find significant metallicity gradients with galactocentric distance, and with distance from the galactic mid-plane.

(2) The non-parametric regression suggests that the dependence of metallicity on z and R may not be monotonic.

It is particularly interesting that within the errors the regression function f_R is constant for $R < 9.5$ (Fig. 7; consistent with the discussion at the end of Section 2). The same applies for $R > 12.5$, whereas there is a steep gradient over the interval $10 < R < 12$. This lends support to the contention of a step-like dependence of the metallicity on galactocentric distance. Yong et al. (2005) provide a brief review of proposed explanations for such a functional dependence.

Fig. 8 shows a substantial dip in the metal abundance at a distance of about 300 pc from the galactic plane. This is primarily a result of several low metallicity clusters at $200 < z < 400$ pc – see Fig. 2.

(3) The radial gradient in [Fe/H] is substantially smaller than previously found from open cluster data. There appears to be two main reasons for this result: the first is that the metallicity should not be regressed on R or z separately. The R–z entry in Table 1 shows that there in substantial positive correlation between these two independent variables (see also Fig. 4). The implication is that if

either of these two independent variables is left out of the regression, the magnitude of the derived abundance slopes will be exaggerated. The second reason why substantially different metallicity gradients could be derived in different studies is the dependence of results on the exact data which are included in the analysis – compare, for example equations (2) and (3) above. The point is further illustrated by inspection of Fig. 7, which shows the dependence of [Fe/H] on R in detail. If this relation is to be replaced by a single straight line, then clearly its slope will be sensitive to the inclusion or exclusion of data with $R > 13$ kpc.

(4) Both linear regression, and non-parametric regression, point to a weak *increase* in metallicity with age, at least for $R < 9.5$. Although the conventional view has been that metallicity decreases with age, the processes at work, such as episodic enrichment, infall of metal-poor material into the galaxy, orbital diffusion etc., are complex (see e.g. the discussion in Bensby et al. 2007), and therefore our result is probably not entirely unrealistic.

The reader's attention is drawn also to fig. 1 in Andersen et al. (2005), which contrasts a recent age-abundance estimate for the solar neighbourhood with earlier work. They find a very weak dependence, with pronounced scatter, rather than the strong negative trend previously claimed.

For $R > 9.5$ the abundance depends strongly on R and z, and hence uncovering its age dependence is more difficult.

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