

Probing primordial features with the primary CMB

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ABSTRACT

We propose to study the imprint of features in the primordial power spectrum with the primary CMB after the subtraction of the reconstructed ISW signal from the observed CMB temperature angular power spectrum. We consider the application to features models able to fit two of the large scales anomalies observed in the CMB temperature angular power spectrum: the deficit of power at $\ell \sim 2$ and at $\ell \sim 22$.

We show that if the features comes from the primordial power spectrum we should be find consistent constraints of these features model from the CMB temperature angular power spectrum removing or not the late ISW signal. Moreover, this method shows also some improvement on the constraints on the features parameters up to 16% for models predicting a suppression of power of the quadrupole and up to 27% for models with features at $\ell \sim 22$, assuming instrumental sensitivity similar to the *Planck* satellite (depending on the goodness of the ISW reconstruction). Furthermore, it gives the opportunity to understand if these anomalies are attributed to early- or late-time physics.

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1. Introduction

Although observations show how a spatially flat Λ CDM model with a tilted power-law spectrum of primordial density fluctuations provides a good fit to CMB temperature and polarization anisotropies [1], there are interesting hints for new physics beyond the Λ CDM model based on slow-roll inflation in the WMAP [2] and *Planck* data [3–5], such as anomalies in the large angular scale pattern of CMB temperature anisotropies [2–19].

Anomalies in the CMB angular power spectra, as well as in the dark matter power spectrum, are predicted by several theoretically well motivated mechanisms that occur during inflation; these mechanisms support deviations from a simple power-law for the primordial power spectrum, connected with the violation of the slow-roll phase, and provide a better fit to the CMB data at $\sim 2\sigma$.

In Fig. 1, it is plotted the comparison between the best-fit CMB temperature power spectrum for the standard Λ CDM model and the best-fits for some features models [4] which improve the fit of CMB data. Although the difference between these models, the cosmic-variance restricts our ability to discriminate between them even with a perfect measure of the CMB anisotropies.

The situation improves if well-suited data in addition to the CMB temperature anisotropies are available:

- CMB *E*-mode polarization have been highlighted as a possible way to constrain primordial features with high confidence

thanks to the narrower transfer functions compared to the ones of the CMB temperature [20–23].

- The opportunity to look elsewhere for the imprint of primordial features, as in the matter power spectrum, is a unique chance to improve our current understanding of these possible anomalies; see for instance [24–33].
- Combined search for primordial features in the power spectrum and bispectrum is another promising way to test such models thanks to the imprints on higher-order correlators [29,34–39].

In this paper, we propose a further method to improve the current understanding of the large scales CMB anomalies based on the possibility to subtract the reconstructed integrated Sachs–Wolfe (ISW) signal from the observed CMB temperature angular power spectrum in order to constrain models with features in the primordial power spectrum with the primary CMB. After subtracting the ISW signal, possible by cross-correlating CMB maps with tracer maps of the matter density fluctuation, we have the opportunity to test the CMB angular power spectrum dominated by the SW contribution at the largest scales like the primary CMB signal generated at the last scattering surface. This technique has been already proposed and applied to real data to study the significance of anomalies at CMB maps level with and without the contamination from the late ISW signal [40–43].

ISW, such as CMB lensing deflection, can be considered as foreground contribution to the primary CMB signal. They can be used to further study the information content from some late-time physics (dark energy and small scales matter perturbation for instance),

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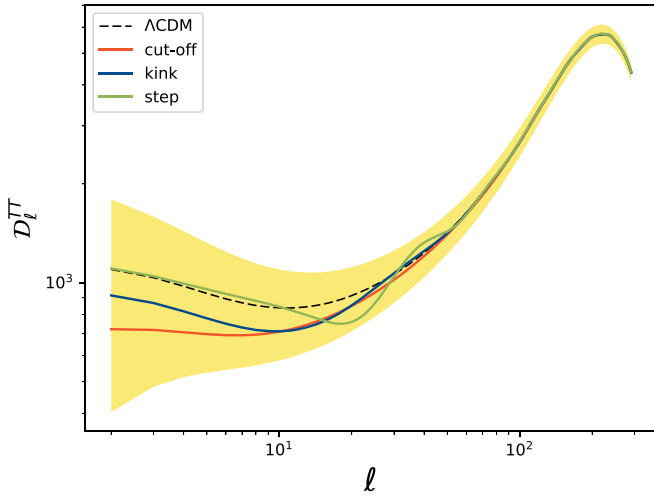


Fig. 1. CMB temperature angular power spectrum best-fit for Λ CDM (dashed black), **cut-off** (red), **kink** (blue), **step** (green) models from *Planck* 2015 TT+lowP data [4]. The yellow band represents the error bar from the cosmic-variance only for Λ CDM. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

but they also ceil part of the primary CMB signal introducing some degeneracies between primordial parameters and other ones.

The ISW contribution to the CMB temperature fluctuations in direction $\hat{\mathbf{n}}$ is a secondary anisotropy in the CMB caused by the passage of CMB photons through evolving gravitational potential wells

$$\frac{\delta T_{\text{ISW}}}{T}(\hat{\mathbf{n}}) = - \int dz e^{-\tau(z)} \left[\frac{d\Phi}{dz}(\hat{\mathbf{n}}, z) + \frac{d\Psi}{dz}(\hat{\mathbf{n}}, z) \right], \quad (1)$$

where Φ and Ψ are the gravitational potentials in the longitudinal gauge and $e^{-\tau(z)}$ is the visibility function. On large scales, in a dark-energy-dominated universe, CMB photons gain energy when they pass through the decaying potential wells associated with overdensities and lose energy on passing through underdensities [44]. This effect mainly contributes to large angular scales and therefore at low multipoles, i.e. $\lesssim 100$, since there is a little power in the potentials at late times on scales that entered the Hubble radius during radiation domination.

2. Primary CMB anisotropies temperature angular power spectrum

The different components that source the observed CMB temperature are assumed to have a negligible correlation with the others and drawn from a Gaussian distributions with mean zero and a known covariance matrix. Therefore, the combination $T^{\text{obs}} - T^{\text{ISW}} = T^{\text{primary}} + T^{\mathcal{N}} + T^{\text{fg}}$ will be distributed as a Gaussian with covariance matrix equals to the sum of the covariance matrices of the noise and the primordial CMB.

By knowing the ISW contribution to the CMB temperature anisotropies it is possible to reconstruct the primary CMB temperature angular power spectrum at large scales

$$C_{\ell}^{\text{primary}} = C_{\ell}^{\text{TT}} - C_{\ell}^{\text{ISW}}, \quad (2)$$

$$\mathcal{N}_{\ell}^{\text{primary}} = \mathcal{N}_{\ell}^{\text{T}} + \mathcal{N}_{\ell}^{\text{ISW}}, \quad (3)$$

where $\mathcal{N}_{\ell}^{\text{ISW}}$ is the noise of the ISW angular power spectrum after the reconstruction.

ISW is generally reconstructed by cross-correlating CMB temperature angular power spectrum with LSS galaxy surveys [45]

or other LSS tracers such as CMB lensing [46], thermal Sunyaev-Zeldovich [47], intensity mapping emission lines [48] and clusters of galaxies [49].

In order to quantify the errors from the reconstruction of the ISW signal by cross-correlating the CMB with one or more LSS tracers, we consider the standard theoretical signal-to-noise ratio (defined according to [50,51]) to build the noise angular power spectra for three different cases: a 3σ level reconstruction of the ISW signal, compatible with the significance obtained in [52] by cross-correlating the *Planck* temperature map with a compilation of publicly available galaxy surveys [52]; a 6σ significance expected for next-generation of LSS galaxy surveys [48,53]; an ideal case with a perfect reconstruction ($\sim 10\sigma$) of the late-time ISW signal with $\mathcal{N}^{\text{ISW}} \simeq 0$ in Eq. (3).

3. Fisher forecast formalism

With these definitions in hand, we can proceed to perform a Fisher matrix analysis for CMB angular power spectra (temperature and E-mode polarization) [54–58]

$$\mathcal{F}_{\alpha\beta}^{\text{CMB}} = \frac{1}{2} \text{tr} [\mathbf{C}_{,\alpha} \mathbf{C}^{-1} \mathbf{C}_{,\beta} \mathbf{C}^{-1}], \quad (4)$$

where

$$\mathbf{C} = \begin{bmatrix} \tilde{C}_{\ell}^{\text{TT}} & \tilde{C}_{\ell}^{\text{TE}} \\ \tilde{C}_{\ell}^{\text{TE}} & \tilde{C}_{\ell}^{\text{EE}} \end{bmatrix}. \quad (5)$$

Here \tilde{C}_{ℓ}^X is the sum of the theoretical spectrum C_{ℓ}^X and the effective noise \mathcal{N}_{ℓ}^X , which is given by the inverse noise weighted combination of the instrumental noise de-convolved with the beams of different frequency channels. For the temperature and polarization angular power spectra, a noise power spectrum with Gaussian beam profile [54] has been used

$$\mathcal{N}_{\ell}^X = \sigma_X b_{\ell}^{-2}. \quad (6)$$

Here b_{ℓ}^2 is the beam window function, assumed Gaussian, with $b_{\ell} = e^{-\ell(\ell+1)\theta_{\text{FWHM}}^2/16 \ln 2}$; θ_{FWHM} is the full width half maximum (FWHM) of the beam in radians; σ_{T} and σ_{E} are the square of the detector noise level on a steradian patch for temperature and polarization, respectively.

4. Models of features in the primordial power spectrum

We consider three inflation models that generate features in the primordial power spectrum (see Fig. 1): the **cut-off** model [59] which reproduces a suppression of power at large scales, and two models which lead to localized features in the primordial power spectrum, i.e. the **kink** model [60] and the **step** model [61]. Following [28], the fiducial spectra are centred at their best-fit parameters from *Planck* 2015 TT+lowP data [4] for each parameterization. The primordial power spectrum can be written as the standard power-law $\mathcal{P}_{\mathcal{R},0}$, modulated by the contribution dues to the violation of slow-roll

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R},0}(k) \cdot \mathcal{P}_{\mathcal{R},\chi}(k), \quad (7)$$

$$\mathcal{P}_{\mathcal{R},0}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1}, \quad (8)$$

where A_s is the amplitude of the curvature power spectrum, n_s is the scalar spectral index and the pivot scale is fixed at $k_* = 0.05 \text{ Mpc}^{-1}$.

The non-canonical contribution to $\mathcal{P}_{\mathcal{R}}$ for the **cutoff** model is given by

$$\mathcal{P}_{\mathcal{R},\text{cutoff}}(y) = 1 - e^{-y^{\lambda c}}, \quad (9)$$

$$y \equiv \frac{k}{k_c}, \quad (10)$$

for the **kink** model by

$$\begin{aligned} \mathcal{P}_{\mathcal{R}, \text{kink}}(y) &= 1 + \frac{9}{2} \mathcal{A}_{\text{kink}}^2 \left(\frac{1}{y} + \frac{1}{y^3} \right)^2 \\ &+ \frac{3}{2} \mathcal{A}_{\text{kink}} \left(4 + 3\mathcal{A}_{\text{kink}} - 3 \frac{\mathcal{A}_{\text{kink}}}{y^4} \right)^2 \frac{1}{y^2} \cos(2y) \\ &+ 3\mathcal{A}_{\text{kink}} \left(1 - \frac{1 + 3\mathcal{A}_{\text{kink}}}{y^2} - \frac{3\mathcal{A}_{\text{kink}}}{y^4} \right)^2 \frac{1}{y} \sin(2y), \end{aligned} \quad (11)$$

$$y \equiv \frac{k}{k_{\text{kink}}}, \quad (12)$$

and for the **step** model by

$$\mathcal{P}_{\mathcal{R}, \text{step}}(y) = \exp \left\{ \mathcal{I}_0(y) + \ln \left[1 + \mathcal{I}_1^2(y) \right] \right\}, \quad (13)$$

$$y \equiv \frac{k}{k_{\text{step}}}, \quad (14)$$

where the first- and second-order parts are

$$\mathcal{I}_0(y) = \mathcal{A}_{\text{step}} W'(y) \mathcal{D} \left(\frac{y}{x_{\text{step}}} \right), \quad (15)$$

$$\sqrt{2} \mathcal{I}_1(y) = \frac{\pi}{2} (1 - n_s) + \mathcal{A}_{\text{step}} X'(y) \mathcal{D} \left(\frac{y}{x_{\text{step}}} \right), \quad (16)$$

where a prime denotes $d/d \ln y$, and the damping envelope is

$$\mathcal{D}(y) = \frac{y}{\sinh y}. \quad (17)$$

The window functions are

$$W(y) = \frac{3 \sin(2y)}{2y^3} - \frac{3 \cos(2y)}{y^2} - \frac{3 \sin(2y)}{2y}, \quad (18)$$

$$X(y) = \frac{3}{y^3} (\sin y - y \cos y)^2. \quad (19)$$

See Refs. [28,59–61] for a clear description of the models.

5. Results

After the ISW removal, the signal of the CMB temperature anisotropies decreases at the large angular scales. On these scales where the instrumental noise is negligible, this effect is compensated by an effective lower cosmic-variance since $\sqrt{2/(2\ell+1)}C_\ell$, i.e.

$$\frac{C_\ell^{\text{TT}}}{C_\ell^{\text{TT}} + \mathcal{N}_\ell^{\text{T}}} \approx \frac{C_\ell^{\text{primary}}}{C_\ell^{\text{primary}} + \mathcal{N}_\ell^{\text{primary}}}, \quad (20)$$

assuming a negligible $\mathcal{N}_\ell^{\text{ISW}}$. On the other hand, the primary CMB is more sensible to the variation of the cosmological parameters connected with the primordial power spectrum. It is possible to see this effect by looking at the derivatives of the CMB temperature anisotropies. In Fig. 2, the derivatives of the CMB primary anisotropies respect to the features parameters are always larger in amplitude compared to the derivatives of the observed CMB temperature anisotropies.

We consider two different configurations of CMB experiment: a representative of current CMB measurements by considering the *Planck* 143 GHz channel full mission sensitivity and angular resolution as given in [62] and a CMB cosmic-variance limited experiment, both with $f_{\text{sky}} = 0.7$. Results are collected in Table 1.

Assuming a perfect reconstruction of the ISW, we found that for the **cut-off** model the errors decrease by 6% on λ_c and by 16% on $\log_{10}(k_c \text{ Mpc}^{-1})$ for an experiment with *Planck*'s sensitivity. For the **kink** model the errors improve by a 17% on $\mathcal{A}_{\text{kink}}$ and by 10% on $\log_{10}(k_{\text{kink}} \text{ Mpc}^{-1})$. The **step** model is the one that benefits more

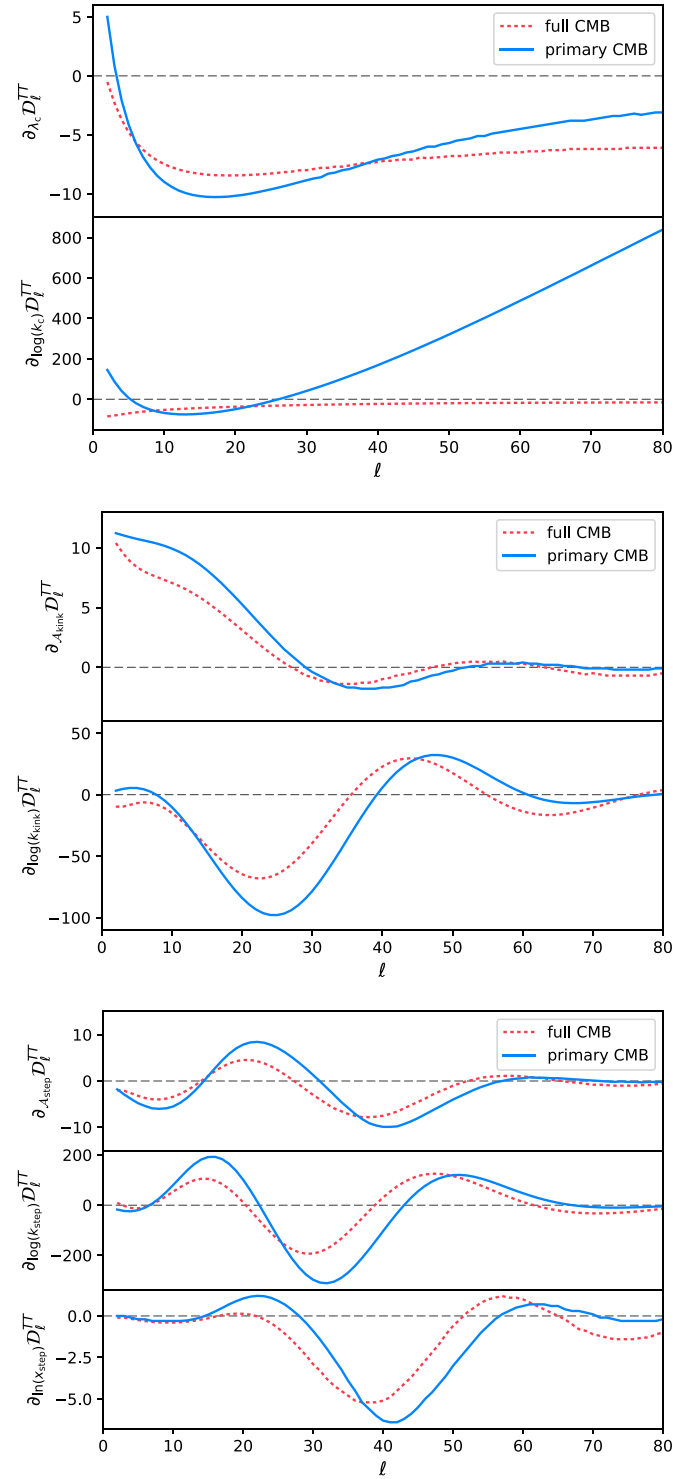


Fig. 2. Derivatives of the CMB temperature angular power spectrum with respect to the features parameters for the **cut-off** (top panels), **kink** (central panels), **step** (bottom panels) models. The red dashed lines refer to the derivative of the full observed CMB temperature angular power spectrum and the blue solid lines refer to the derivative of the primary spectra. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

from this method, the errors improve by a 27% on $\mathcal{A}_{\text{step}}$, by 25% on $\log_{10}(k_{\text{step}} \text{ Mpc}^{-1})$ and by 19% on $\ln(x_{\text{step}})$.

The case with the subtraction of the 3σ and of the 6σ detected ISW does not lead to any improvements for both the **cut-off** and the **kink** models. Instead, for the **step** model there is still a

Table 1

68% constraints on the features parameters for a *Planck*-like CMB experiment (left) and a cosmic-variance limited one (right). Constraints are given for the standard case (full CMB) and after ISW subtraction by considering different levels of ISW detection. We report the best-fit for the features parameters from *Planck* 2015 TT + lowP data [4].

Model	Parameters	Full CMB	3 σ ISW	6 σ ISW	Primary CMB
cut-off	$\lambda_c = 0.50$	0.218/0.176	0.340/0.234	0.264/0.196	0.204/0.162
	$\log_{10}(k_c \text{ Mpc}^{-1}) = -3.47$	0.371/0.325	0.564/0.440	0.418/0.357	0.310/0.280
kink	$\mathcal{A}_{\text{kink}} = 0.089$	0.0466/0.0334	0.0779/0.0430	0.534/0.0360	0.0387/0.0296
	$\log_{10}(k_{\text{kink}} \text{ Mpc}^{-1}) = -3.05$	0.0962/0.0530	0.129/0.0564	0.105/0.0549	0.0866/0.0529
step	$\mathcal{A}_{\text{step}} = 0.374$	0.257/0.125	0.344/0.130	0.247/0.126	0.187/0.120
	$\log_{10}(k_{\text{step}} \text{ Mpc}^{-1}) = -3.1$	0.0368/0.0165	0.0418/0.0174	0.0336/0.0168	0.0275/0.0160
	$\ln(x_{\text{step}}) = 0.342$	0.362/0.189	0.471/0.199	0.364/0.192	0.293/0.183

reduced improvement of 5% on the amplitude and 10% on the scale parameter, even for these cases with injected noise from the ISW reconstruction.

Feature models which predict departures from the standard power-law primordial power spectrum will benefit from having better measurements of large angular scale CMB *E*-mode polarization at the cosmic-variance level. However, the **cut-off** model affects the largest angular scales reproducing a suppression of power at $\ell < 30$ in temperature and $\ell < 10$ in the *E*-mode polarization. For this reason, the relative improvement does not change when we consider a cosmic-variance limited CMB experiment. The instrumental noise on the *E*-mode polarization is small even for *Planck* on such scales. In this case the improvement from the subtraction of the ISW signal is very small, $\sim 5\%$, even for the case of perfect ISW reconstruction for all the three considered models.

6. Conclusion

In conclusion, this method represent a consistent way to test the origin of features in the CMB temperature angular power spectrum.

We show that the constraints on features parameters after the ISW subtraction are expected to be consistent with the ones obtained from full CMB. Moreover, this approach performs well for the **step** model which fits the deficit in power at $\ell \simeq 20 - 30$, improving the constraints by 5 – 27% on the amplitude and by 10 – 25% on the scale parameters, even without better measurements of CMB polarization.

Finally, even if the final improvement for realistic cases of ISW subtraction could lead to small differences in terms of constraining power on the parameters of these features models, the subtraction of the ISW signal could lead to a change in the pattern of the largest scales of the CMB temperature anisotropies changing the shape of the features. For instance, if an anomaly vanishes after the subtraction of the ISW component to the CMB temperature, then a primordial explanation would be eliminated.

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References

- [1] Y. Akrami, et al., [Planck Collaboration], [arXiv:1807.06205](https://arxiv.org/abs/1807.06205) [astro-ph.CO].
- [2] H.V. Peiris, et al., [WMAP Collaboration], *Astrophys. J. Suppl.* 148 (2003) 213, <http://dx.doi.org/10.1086/377228> [astro-ph/0302225].

- [3] P.A.R. Ade, et al., [Planck Collaboration], *Astron. Astrophys.* 571 (2014) A22, <http://dx.doi.org/10.1051/0004-6361/201321569>, [arXiv:1303.5082](https://arxiv.org/abs/1303.5082) [astro-ph.CO].
- [4] P.A.R. Ade, et al., [Planck Collaboration], *Astron. Astrophys.* 594 (2016) A20, <http://dx.doi.org/10.1051/0004-6361/201525898>, [arXiv:1502.02114](https://arxiv.org/abs/1502.02114) [astro-ph.CO].
- [5] Y. Akrami, et al., [Planck Collaboration], [arXiv:1807.06211](https://arxiv.org/abs/1807.06211) [astro-ph.CO].
- [6] L. Covi, J. Hamann, A. Melchiorri, A. Slosar, I. Sorbera, *Phys. Rev. D* 74 (2006) 083509, <http://dx.doi.org/10.1103/PhysRevD.74.083509> [astro-ph/0606452].
- [7] M. Benetti, *Phys. Rev. D* 88 (2013) 087302, <http://dx.doi.org/10.1103/PhysRevD.88.087302>, [arXiv:1308.6406](https://arxiv.org/abs/1308.6406) [astro-ph.CO].
- [8] V. Miranda, W. Hu, *Phys. Rev. D* 89 (8) (2014) 083529, <http://dx.doi.org/10.1103/PhysRevD.89.083529>, [arXiv:1312.0946](https://arxiv.org/abs/1312.0946) [astro-ph.CO].
- [9] R. Easther, R. Flauger, *J. Cosmol. Astropart. Phys.* 1402 (2014) 037, <http://dx.doi.org/10.1088/1475-7516/2014/02/037>, [arXiv:1308.3736](https://arxiv.org/abs/1308.3736) [astro-ph.CO].
- [10] X. Chen, M.H. Namjoo, *Phys. Lett. B* 739 (2014) 285, <http://dx.doi.org/10.1016/j.physletb.2014.11.002>, [arXiv:1404.1536](https://arxiv.org/abs/1404.1536) [astro-ph.CO].
- [11] A. Achucarro, V. Atal, B. Hu, P. Ortiz, J. Torrado, *Phys. Rev. D* 90 (2) (2014) 023511, <http://dx.doi.org/10.1103/PhysRevD.90.023511>, [arXiv:1404.7522](https://arxiv.org/abs/1404.7522) [astro-ph.CO].
- [12] D.K. Hazra, A. Shafieloo, G.F. Smoot, A.A. Starobinsky, *J. Cosmol. Astropart. Phys.* 1408 (2014) 048, <http://dx.doi.org/10.1088/1475-7516/2014/08/048>, [arXiv:1405.2012](https://arxiv.org/abs/1405.2012) [astro-ph.CO].
- [13] D.K. Hazra, A. Shafieloo, T. Souradeep, *J. Cosmol. Astropart. Phys.* 1411 (11) (2014) 011, <http://dx.doi.org/10.1088/1475-7516/2014/11/011>, [arXiv:1406.4827](https://arxiv.org/abs/1406.4827) [astro-ph.CO].
- [14] B. Hu, J. Torrado, *Phys. Rev. D* 91 (6) (2015) 064039, <http://dx.doi.org/10.1103/PhysRevD.91.064039>, [arXiv:1410.4804](https://arxiv.org/abs/1410.4804) [astro-ph.CO].
- [15] A. Gruppuso, A. Sagnotti, *Internat. J. Modern Phys. D* 24 (12) (2015) 1544008, <http://dx.doi.org/10.1142/S0218271815440083>, [arXiv:1506.08093](https://arxiv.org/abs/1506.08093) [astro-ph.CO].
- [16] A. Gruppuso, N. Kitazawa, N. Mandolesi, P. Natoli, A. Sagnotti, *Phys. Dark Univ.* 11 (2016) 68, <http://dx.doi.org/10.1016/j.dark.2015.12.001>, [arXiv:1508.00411](https://arxiv.org/abs/1508.00411) [astro-ph.CO].
- [17] D.K. Hazra, A. Shafieloo, G.F. Smoot, A.A. Starobinsky, *J. Cosmol. Astropart. Phys.* 1609 (09) (2016) 009, <http://dx.doi.org/10.1088/1475-7516/2016/09/009>, [arXiv:1605.02106](https://arxiv.org/abs/1605.02106) [astro-ph.CO].
- [18] J. Torrado, B. Hu, A. Achucarro, *Phys. Rev. D* 96 (8) (2017) 083515, <http://dx.doi.org/10.1103/PhysRevD.96.083515>, [arXiv:1611.10350](https://arxiv.org/abs/1611.10350) [astro-ph.CO].
- [19] G. Obied, C. Dvorkin, C. Heinrich, W. Hu, V. Miranda, *Phys. Rev. D* 98 (4) (2018) 043518, <http://dx.doi.org/10.1103/PhysRevD.98.043518>, [arXiv:1803.01858](https://arxiv.org/abs/1803.01858) [astro-ph.CO].
- [20] M.J. Mortonson, C. Dvorkin, H.V. Peiris, W. Hu, *Phys. Rev. D* 79 (2009) 103519, <http://dx.doi.org/10.1103/PhysRevD.79.103519>, [arXiv:0903.4920](https://arxiv.org/abs/0903.4920) [astro-ph.CO].
- [21] J. Chluba, J. Hamann, S.P. Patil, *Internat. J. Modern Phys. D* 24 (10) (2015) 1530023, <http://dx.doi.org/10.1142/S0218271815300232>, [arXiv:1505.01834](https://arxiv.org/abs/1505.01834) [astro-ph.CO].
- [22] F. Finelli, et al., [CORE Collaboration], *J. Cosmol. Astropart. Phys.* 1804 (2018) 016, <http://dx.doi.org/10.1088/1475-7516/2018/04/016>, [arXiv:161208270](https://arxiv.org/abs/161208270) [astro-ph.CO].
- [23] D.K. Hazra, D. Paoletti, M. Ballardini, F. Finelli, A. Shafieloo, G.F. Smoot, A.A. Starobinsky, *J. Cosmol. Astropart. Phys.* 1802 (02) (2018) 017, <http://dx.doi.org/10.1088/1475-7516/2018/02/017>, [arXiv:171001205](https://arxiv.org/abs/171001205) [astro-ph.CO].
- [24] H. Zhan, L. Knox, A. Tyson, V. Margoniner, *Astrophys. J.* 640 (2006) 8, <http://dx.doi.org/10.1086/500077> [astro-ph/0508119].
- [25] Z. Huang, L. Verde, F. Vernizzi, *J. Cosmol. Astropart. Phys.* 1204 (2012) 005, <http://dx.doi.org/10.1088/1475-7516/2012/04/005>, [arXiv:12015955](https://arxiv.org/abs/12015955) [astro-ph.CO].
- [26] X. Chen, C. Dvorkin, Z. Huang, M.H. Namjoo, L. Verde, *J. Cosmol. Astropart. Phys.* 1611 (11) (2016) 014, <http://dx.doi.org/10.1088/1475-7516/2016/11/014>, [arXiv:1605.09365](https://arxiv.org/abs/1605.09365) [astro-ph.CO].
- [27] X. Chen, P.D. Meerburg, M. Münchmeyer, *J. Cosmol. Astropart. Phys.* 1609 (09) (2016) 023, <http://dx.doi.org/10.1088/1475-7516/2016/09/023>, [arXiv:1605.09364](https://arxiv.org/abs/1605.09364) [astro-ph.CO].

- [28] M. Ballardini, F. Finelli, C. Fedeli, L. Moscardini, J. Cosmol. Astropart. Phys. 1610 (2016) 041, J. Cosmol. Astropart. Phys. 1804 (04) (2018) E01 (erratum). <http://dx.doi.org/10.1088/1475-7516/2018/04/E01>, <http://dx.doi.org/10.1088/1475-7516/2016/10/041>, arXiv:160603747 [astro-ph.CO].
- [29] Y. Xu, J. Hamann, X. Chen, Phys. Rev. D 94 (12) (2016) 123518, <http://dx.doi.org/10.1103/PhysRevD.94.123518>, arXiv:1607.00817 [astro-ph.CO].
- [30] M.A. Fard, S. Baghrani, J. Cosmol. Astropart. Phys. 1801 (01) (2018) 051, <http://dx.doi.org/10.1088/1475-7516/2018/01/051>, arXiv:170905323 [astro-ph.CO].
- [31] G.A. Palma, D. Sapone, S. Sypsas, J. Cosmol. Astropart. Phys. 1806 (06) (2018) 004, <http://dx.doi.org/10.1088/1475-7516/2018/06/004>, arXiv:171002570 [astro-ph.CO].
- [32] B. L'Huillier, A. Shafieloo, D.K. Hazra, G.F. Smoot, A.A. Starobinsky, Mon. Not. R. Astron. Soc. 477 (2) (2018) 2503, <http://dx.doi.org/10.1093/mnras/sty745>, arXiv:1710.10987 [astro-ph.CO].
- [33] M. Ballardini, F. Finelli, R. Maartens, L. Moscardini, J. Cosmol. Astropart. Phys. 1804 (04) (2018) 044, <http://dx.doi.org/10.1088/1475-7516/2018/04/044>, arXiv:171207425 [astro-ph.CO].
- [34] J.R. Fergusson, H.F. Gruetjen, E.P.S. Shellard, M. Liguori, Phys. Rev. D 91 (2) (2015) 023502, <http://dx.doi.org/10.1103/PhysRevD.91.023502>, arXiv:1410.5114 [astro-ph.CO].
- [35] J.R. Fergusson, H.F. Gruetjen, E.P.S. Shellard, B. Wallisch, Phys. Rev. D 91 (12) (2015) 123506, <http://dx.doi.org/10.1103/PhysRevD.91.123506>, arXiv:1412.6152 [astro-ph.CO].
- [36] P.A.R. Ade, et al., [Planck Collaboration], Astron. Astrophys. 594 (2016) A17, <http://dx.doi.org/10.1051/0004-6361/201525836>, arXiv:1502.01592 [astro-ph.CO].
- [37] P.D. Meerburg, M. Münchmeyer, B. Wandelt, Phys. Rev. D 93 (4) (2016) 043536, <http://dx.doi.org/10.1103/PhysRevD.93.043536>, arXiv:1510.01756 [astro-ph.CO].
- [38] A. Moradinezhad Dizgah, H. Lee, J.B. Muoz, C. Dvorkin, J. Cosmol. Astropart. Phys. 1805 (05) (2018) 013, <http://dx.doi.org/10.1088/1475-7516/2018/05/013>, arXiv:180107265 [astro-ph.CO].
- [39] D. Karagiannis, A. Lazanu, M. Liguori, A. Raccanelli, N. Bartolo, L. Verde, Mon. Not. R. Astron. Soc. 478 (1) (2018) 1341, <http://dx.doi.org/10.1093/mnras/sty1029>, arXiv:1801.09280 [astro-ph.CO].
- [40] G. Efstathiou, Y.Z. Ma, D. Hanson, Mon. Not. R. Astron. Soc. 407 (2010) 2530, <http://dx.doi.org/10.1111/j.1365-2966.2010.17081.x>, arXiv:0911.5399 [astro-ph.CO].
- [41] C.L. Francis, J.A. Peacock, Mon. Not. R. Astron. Soc. 406 (2010) 14, <http://dx.doi.org/10.1111/j.1365-2966.2010.16866.x>, arXiv:0909.2495 [astro-ph.CO].
- [42] J. Muir, D. Huterer, Phys. Rev. D 94 (4) (2016) 043503, <http://dx.doi.org/10.1103/PhysRevD.94.043503>, arXiv:1603.06586 [astro-ph.CO].
- [43] C.J. Copi, M. O'Dwyer, G.D. Starkman, Mon. Not. R. Astron. Soc. 463 (3) (2016) 3305, <http://dx.doi.org/10.1093/mnras/stw2163>, arXiv:1605.09732 [astro-ph.CO].
- [44] L. Kofman, A.A. Starobinsky, Sov. Astron. Lett. 11 (1985) 271; Pisma Astron. Zh. 11 (1985) 643.
- [45] R.G. Crittenden, N. Turok, Phys. Rev. Lett. 76 (1996) 575, <http://dx.doi.org/10.1103/PhysRevLett.76.575> [astro-ph/951007].
- [46] A. Manzotti, S. Dodelson, Phys. Rev. D 90 (12) (2014) 123009, <http://dx.doi.org/10.1103/PhysRevD.90.123009>, arXiv:1407.5623 [astro-ph.CO].
- [47] N. Taburet, C. Hernandez-Monteagudo, N. Aghanim, M. Douspis, R.A. Sunyaev, Mon. Not. R. Astron. Soc. 418 (2011) 2207, <http://dx.doi.org/10.1111/j.1365-2966.2011.19474.x>, arXiv:10125036 [astro-ph.CO].
- [48] A. Pourtsidou, D. Bacon, R. Crittenden, Mon. Not. R. Astron. Soc. 470 (4) (2017) 4251, <http://dx.doi.org/10.1093/mnras/stx1479>, arXiv:1610.04189 [astro-ph.CO].
- [49] M. Ballardini, D. Paoletti, F. Finelli, L. Moscardini, B. Sartoris, L. Valenziano, arXiv:1712.02380 [astro-ph.CO].
- [50] A. Cooray, Phys. Rev. D 65 (2002) 103510, <http://dx.doi.org/10.1103/PhysRevD.65.103510> [astro-ph/0112408].
- [51] N. Afshordi, Phys. Rev. D 70 (2004) 083536, <http://dx.doi.org/10.1103/PhysRevD.70.083536> [astro-ph/0401166].
- [52] P.A.R. Ade, et al., [Planck Collaboration], Astron. Astrophys. 594 (2016) A21, <http://dx.doi.org/10.1051/0004-6361/201525831>, arXiv:1502.01595 [astro-ph.CO].
- [53] A. Raccanelli, E. Kovetz, L. Dai, M. Kamionkowski, Phys. Rev. D 93 (8) (2016) 083512, <http://dx.doi.org/10.1103/PhysRevD.93.083512>, arXiv:1502.03107 [astro-ph.CO].
- [54] L. Knox, Phys. Rev. D 52 (1995) 4307, <http://dx.doi.org/10.1103/PhysRevD.52.4307> [astro-ph/9504054].
- [55] G. Jungman, M. Kamionkowski, A. Kosowsky, D.N. Spergel, Phys. Rev. D 54 (1996) 1332, <http://dx.doi.org/10.1103/PhysRevD.54.1332> [astro-ph/9512139].
- [56] U. Seljak, Astrophys. J. 482 (6) (1997) <http://dx.doi.org/10.1086/304123> [astro-ph/9608131].
- [57] M. Zaldarriaga, U. Seljak, Phys. Rev. D 55 (1997) 1830, <http://dx.doi.org/10.1103/PhysRevD.55.1830>, [astro-ph/9609170].
- [58] M. Kamionkowski, A. Kosowsky, A. Stebbins, Phys. Rev. D 55 (1997) 7368, <http://dx.doi.org/10.1103/PhysRevD.55.7368> [astro-ph/9611125].
- [59] C.R. Contaldi, M. Peloso, L. Kofman, A.D. Linde, J. Cosmol. Astropart. Phys. 0307 (2003) 002, <http://dx.doi.org/10.1088/1475-7516/2003/07/002> [astro-ph/0303636].
- [60] A.A. Starobinsky, JETP Lett. 55 (1992) 489; Pis'ma Zh. Eksp. Teor. Fiz. 55 (1992) 477.
- [61] C. Dvorkin, W. Hu, Phys. Rev. D 81 (2010) 023518, <http://dx.doi.org/10.1103/PhysRevD.81.023518>, arXiv:0910.2237 [astro-ph.CO].
- [62] R. Adam, et al., [Planck Collaboration], Astron. Astrophys. 594 (2016) A1, <http://dx.doi.org/10.1051/0004-6361/201527101>, arXiv:1502.01582 [astro-ph.CO].