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Values in preservice mathematics teachers' discussions of the Body Mass Index - A critical perspective

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ABSTRACT

This article explores the values that come to the fore when preservice mathematics teachers (PTs) [1] engage in critical discussions about the role of mathematical models in society. The specific model that was discussed was the Body Mass Index (BMI) [2]. From the analysis of the PTs' discussions of the BMI from a mathematical and societal point of view several mathematical and mathematics educational values were identified such as openness, rationalism, progress, reasoning, evaluating, and problematizing the instrumental understanding of mathematics. In addition, critical thinking about mathematics in society as emphasized in curricula in the three countries involved in the study, was identified with four categories of complementary pairs. Knowing the mathematical and mathematics educational values underpinning PTs' discussions and their connection to critical thinking is important for successfully engaging with the role of mathematics in society.

1. Introduction

Values have gained an important place in mathematics education research as their relevance for the teaching and learning of mathematics is long argued for (e.g., Bishop, 1988; Clarkson et al., 2019). The national school curricula in the three countries involved in this study build implicitly or explicitly upon values. The overarching part of the curriculum in Norway builds upon values such as critical thinking and ethical awareness etc. (Ministry of Education, 2020). Similarly, the Albanian curriculum states that education should give students the possibilities to develop values connected to the democratic development, such as citizenship, ethical values, critical thinking etc. (Ministry of Education and Sports, 2014). The curriculum and assessment policy in South Africa emphasizes democratic values and social justice, aiming at critical and active learning, social transformation, human rights, and inclusion and valuing of indigenous knowledge (Department of Basic Education, 2011).

Critical mathematical competence, defined as the ability to use and reflect on mathematics in different contexts, as well as discuss its role in society (Skovsmose, 2016), is a central competence in mathematics curricula. The role of mathematics in society, and critical discussions about it, are at the core of our study. In the three countries involved in the study, we find several aspects of the connection between mathematics and society in the curricula: in South-Africa students need to be critically aware of such connections (Department of Basic Education, 2011); in Norway students need to make their own choices and take a stand on important issues in their lives

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[1] In the remaining parts of the text, we will refer to preservice teachers as PTs.

[2] From now on, the Body Mass Index will be referred to as BMI and is calculated as mass (m) divided by the square of height (h).

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and in society (Ministry of Education, 2020); and in Albania, students should be able to critically use and interpret mathematics in a variety of contexts (Ministry of Education and Sports, 2014). Research is required for ways to support teachers in implementing social issues in mathematics teaching (Mamolo, 2018). Mathematical modelling researchers highlight its relevance for realizing the relationships between mathematics and the world we live in. Making this relationship visible serves, according to Niss (2007), to both enhance knowledge and use that knowledge in further improving and advancing social conditions. Mathematical modelling is also found in the mathematical curricula in the three countries in the study; in Albania, mathematical modelling is seen as one of the six competences in mathematics education (Ministry of Education and Sports, 2014); in Norway mathematical modelling and applications represent one of the six core elements in the mathematics curriculum (Ministry of Education, 2020) focused at understanding the patterns and relationships within society and nature; and in South Africa mathematical modelling is a pivotal point and real-life issues related to health, economic, cultural, scientific, political and environmental issues are specifically mentioned as important (Department of Basic Education, 2011). One important aspect of the link between mathematics and society is the development of a critical stance towards mathematical models and their use and impact on social issues. The latter corresponds to the socio-critical perspective in modelling (Barbosa, 2006; Kaiser & Sriraman, 2006) which is the perspective embraced in the study presented here.

This study focuses on the role of mathematics in society. If we want PTs to engage in teaching mathematics while focusing on the role of mathematics in society, they need to both get experience with such examples and to think that it is worthy to spend time on. As Mamolo (2018) demonstrated in her study, when the PTs engaged as learners with social justice tasks in mathematics, they became aware of the relevance of those kinds of tasks for exposing their students to social issues and connecting mathematics to real-life. In particular, the need for PTs to experience mathematical modelling in order to implement it in their own teaching is argued for in many research articles (see for example Biembengut & Hein, 2010; Borromeo Ferri & Blum, 2010; Maass & Engeln, 2018; Niss, Blum et al., 2007). At the same time, research on values emphasizes that changing mathematics teaching implicitly requires changing the values being taught (Carr, 2019). One good reason for focusing on values is its affective dimension and its impact on teachers' and students' choices of things as worthy to invest time in (Seah et al., 2016). Values are viewed as "guiding our valuing of what is important to students and teachers (and researchers) or not, what is good or bad, what is beautiful or ugly, and so on" (Andersson & Österling, 2019, p. 71). As such, values might change (see e.g. Zhang, 2019). In a study by Baba and Shimada (2019), for example, students changed their mathematical models when they were asked to consider social implications and values. In the context of this article dealing with a mathematical model, the BMI, it is accepted the PTs might change their values due to the mathematical models always being open to revision as work on such models progresses.

From the studies of Hattori et al. (2021) about social values, when students solved socially open-ended problems in Japan, and of Andersson and Österling (2019) in Sweden, about the conflicts between democratic values in the curriculum and the students' mathematical values, it seems that the teachers have an important role in focusing on social values in mathematics and in helping students to develop those required by the curriculum. Bishop (1999) claims that students, thus PTs as well, need to have choices in the classroom and that by looking at the choices students make in different situations one can indicate certain values. When teachers teach mathematics, they make choices guided by their own values, and they do explicitly or implicitly teach values.

In the study presented here, PTs were asked to engage with a task dealing with the BMI and its role in society. The BMI was chosen, as an example of a known and controversial extant mathematical model to be looked upon critically. Earlier it was found that when discussing such a task the PTs took a student position, a citizen position and a teacher position accompanied with the respective kinds of discussions and wordings (Rangnes et al., 2018). This way of studying values by focusing on PTs' discussions of mathematics in society is different from previous studies who mostly use self-reported questionnaires (Carr, 2019). Given the relevance of values as highlighted in the research presented here, we pursued the research question: *what kinds of values do PTs from three different countries exhibit when participating in critical discussions about the mathematical and societal aspects of the BMI and its hypothetical use in mathematics teaching*?

In the following section, we present and discuss theoretical definitions of different kinds of values and their connection to critical mathematics ideas.

2. Theoretical considerations: values and critical perspectives

Values are seen as a variable of affect and Bishop (1999) has emphasized that mathematics teachers do implicitly teach values. Several studies have argued about the values' relevance and impact in mathematics education (see e.g. Bishop et al., 2003; Lim & Ernest, 1997; Seah, 2019 etc.). We will focus at two of the kinds of values in the mathematics classroom that have been considered by Bishop and colleagues: mathematical values and mathematics educational values. The mathematical values have to do with the nature of mathematical knowledge and "are derived from the way mathematicians of different cultures have developed the discipline of mathematics" (Bishop et al., 1999, p. 7). It was Bishop (1988) who first introduced mathematical values in the literature and as Clarkson et al. (2019) noted, since then nothing was added to the three pairs of complementary mathematical values: the ideological values of rationalism and objectism, the sentimental values of control and progress, and the sociological values of openness and mystery. *Rationalism* is connected to deductive reasoning, argument, hypothesis making and logical thinking, while *objectism* relies on symbolizing, representations and on treating abstract ideas as objects. In the second pair, *control* is the feeling of certainty and power one has when mastering the rules of mathematics and applying them also on social problems, while *progress* points to abstracting, generalizing and to changes and growth through questioning. The sentimental value of *openness* focuses on the individual freedom to exposing ideas and arguments in a transparent way, and critically analyzing them to establish both validity and weaknesses of arguments; the last value is connected to the *mystery* of mathematics which attracts and makes people wonder.

The progress in research about values in mathematics education resulted in the Third Wave project focusing on cross-cultural survey studies of what students value in effective mathematics learning (Seah & Peng, 2012). Another relevant type of values for our study are the mathematics educational values that "are expressed through the pedagogical practices of the subject in schools" (Seah, 2013, p. 194). In the Third Wave project, mathematics educational values were identified as the continuum between: ability and effort, wellbeing and hardship, process and product, application and computation, facts and ideas, exposition and exploration, recalling and creating, ICT and pen-and-paper (Seah, 2013). However, for our study, five complementary pairs earlier identified by Seah and Bishop (2000) were found relevant. The pairs are: 1)) *formalistic*, where mathematics learning is receptive and based on deductive reasoning vs *activistic* defined by inductive reasoning and discovery learning; 2) the *instrumental* vs *relational* understanding of mathematics as connected to Skemp (1976) definitions; 3) the *relevance* vs *theoretical* knowledge continuum which is connected to the nature of mathematics learning as being relevant in solving real problems or as being theoretically learned/taught without connections to reality; 4) the *accessibility*, in the sense of mathematics for all vs *specialism* in the sense of mathematics being for a few elite persons; 5) and the last pair is connected to why mathematics is learned and goes from *evaluating* unknown answers to using mathematics for *reasoning* and communicating ideas. One can see some overlap between Seah (2013) values and Seah and Bishop (2000), as e.g., ability and effort can be seen in connection to accessibility, for all those who put effort into learning mathematics, or specialism, for those who possess the ability. However, the reason why the latter values are relevant for our study is that they clearly point to the difference between the "traditional" versus the critical mathematics classroom in terms of how mathematics education is present and what the focus is on those classrooms. All the eight pairs of values as used here, were also used by Dede (2011) in a survey study about Turkish PTs' values about mathematics and mathematics education. Dede found that not all the values identified by Bishop and colleagues were accepted by Turkish PTs, arguing about cultural differences in values. The *evaluating- reasoning* complementary pair did not result as important. As well, some of the values were grouped as belonging together such as for example *relevance*, *activistic* view and *relational* knowledge.

The kinds of values to be found in the mathematics classroom intersect each other and can overlap sometimes. Further, Seah and Bishop (2000) argue about other values outside the mathematics classroom that influence those inside. This is in line with Ernest (2016) who argues that the aims for teaching mathematics express values and cannot be seen in isolation, but together with the "educational and social values of society or some part of it" (p. 3). An example of such values are the ones expressed in the mathematics curricula. Although the three national curricula build upon values such as critical thinking, it is not clear how they are defined in the curricula. We thus refer to *critical thinking* in mathematics education as defined by Jablonka (2020) as including reflections upon the methodological standards of mathematical models, questioning of those standards, but also reflections upon the contexts in which mathematical models are used. Critical thinking thus is closely seen in connection to mathematics in society.

In a critical perspective, there is no such thing as value-free mathematics (education). Ernest (2016) talks about the degradation of the human spirit by pointing attention to the use of mathematics to value objects and practices as in "the accountant's balance sheet" (p. 111). In a similar fashion, Fasheh (2012) problematizes the values of greed, power and control that mathematics teaching can instill upon people when focusing on measuring everything in terms of profit. He sees values in our behavior as the axioms in mathematics and calls for using mathematics to study patterns, relations, and connectedness in life, as well as to "discover the values that govern our actions" (p. 97). Both authors point to the need to be critical towards the formatting power of mathematics which is also problematized by Skovsmose (1994) as being crucial in critical mathematics education.

A central term in critical mathematics education is that of mathematics in action, an active constituent of society which "affords us with a perspective" (Yasukawa et al., 2016, p. 82). Mathematics in action is seen in its four dimensions in terms of the mathematics' functions: description, in that mathematical models are used to describe reality; inscription of certain ideologies in that mathematics provides a lens through which we prioritize certain things and use that lens to see what is and what could be; prescribing, in that certain actions or actors are authorized and mathematics is the "tool for calculating what to do" (p. 83); and, subscription, where mathematical script is accepted and legitimized as a tool to make decisions. Further, mathematical modeling is considered as a way to handle the missing links between school mathematics and reality. Skovsmose (2014) sees a mathematical model as re-presenting reality, thus highlighting that there is no such thing as a one-to-one correspondence between a model and reality. Barbosa (2006) goes further to assert that mathematical models are not neutral descriptions of reality pointing to the modeling process' devices as concealed to the general public. In line with the definition of critical thinking by Jablonka (2020), Skovsmose (2014) argued for the need to not only look at mathematical models from a pure mathematical perspective, but to also scrutinize the intentions behind the creation of the models and the interests behind their use. This is also taken up in Barbosa (2006) related to the modeling as critic perspective and the kinds of discussions focusing on these aspects are defined as reflexive discussions. In a study of mathematical modeling tasks from different perspectives, Dede et al. (2021) found that only the socio-critical modeling tasks involved the *objectivism* value; further, the *progress, accessibility* and *relevance* values were present in all of them, and *rationalism* and *theoretical* values in some of the explored tasks, while other educational values such as *social justice* and *humanity* were present in only the socio-critical modeling tasks. One can thus say that socio-critical modeling tasks, as the BMI task also is, have a potential to expose students for all the kinds of mathematical values in the classroom and especially for other educational values such as social justice.

The investigation of existing mathematical models was seen by Skovsmose (2014) as a way to investigate mathematics in action. In this sense, the BMI is one mathematical model that is used in society to measure obesity (Hall & Barwell, 2015) and where the different dimensions of mathematics in action can be observed: the formula provides a lens to see what the situation is, to calculate, and to further decide how to proceed with the situation. This model is an example of the formatting power of mathematics (Skovsmose, 1994) or the power to shape our perceptions of the world (Ernest, 2016), because by using the model one can categorize people and take measures and give prescriptions based on that (Hall & Barwell, 2015).

In a critical perspective, there is a concern on critically addressing mathematics in all its forms and uses, on understanding and changing society, as well as on considering students' and mathematics teachers' interests (Skovsmose, 2016). The PTs' exposed values thus are included in the concerns of critical mathematics education. Values about what they think mathematics is and should be, both in the classroom and in society, what mathematics education is and should be, together with other personal or societal values, underpin the PTs' decisions if and how to work with existing mathematical models that guide our lives, if and how to be critical towards the uses of mathematics in societal contexts, if and how to include aspects of mathematics in society in teaching. Judging from characteristics of critical thinking, this would fit with mathematical educational values such as accessibility for all, since all students should be able to develop critical thinking in mathematics, activistic since students should engage themselves with mathematics, relevant since students should be critical towards mathematics used in society, relational so that students know what is behind models, and reasoning since the mathematical knowledge should be used to support their arguments. However, as we will see in the results, the PTs expressed also other values connected to mathematics in society and critical thinking, which were difficult to locate into the existing values framework.

3. Methods

The data were collected in Norway, South Africa, and Albania, with the first author facilitating the sessions in each of the locations. The study started as a collaboration between the two authors at their respective institutions (Norway and South Africa) and the first author included Albania during her collaboration with her previous place of work. The researchers shared a common interest in mathematical modeling from a critical perspective. Furthermore, although as alluded to above, all three countries espoused similar values in their mathematics curricula but their histories and experiences with democracy differ. Norway has a long history of being a practicing democracy with its accompanying values. South Africa is a recent democracy having to deal with the eradication of the vestiges of apartheid values, and Albania is also a recent democracy after having struggled under a communist dictatorship with its values since World War II. There has not been much explicit focus on values in mathematics education in the three countries, and the PTs in our study have not had any literature on that in their education to the authors' knowledge.

A toolkit (see Appendix I) consisting of literature in critical mathematics education, a lecture in critical mathematics education and indices, and a question sheet to guide group discussions were used in all the three countries. The same question sheet was translated in the three languages, Norwegian, English, and Albanian. In Norway, three groups of 4–5 Norwegian PTs in mathematics education, at their 4th year, participated in 60 min discussions of the question sheet; three groups of 4th year mathematics PTs, a total of 13 students, participated in the discussions in South Africa, and further two groups of 6 PTs each (master level) in mathematics education participated in Albania. All the PTs agreed to being audio recorded for the study. The question sheet asked the PTs to discuss the BMI as a model that has an impact in society in defining the obesity as a phenomenon. There were questions about the mathematics of the formula and alternative models, about the impact of the model in society and about including BMI and other indexes in mathematics teaching.

When discussing BMI, its mathematical and social aspects, by sharing their thoughts and making choices in collaboration with others, the PTs implicitly express their values about mathematics and mathematics education. When asked to answer questions about BMI and other indexes, in their teaching, they invariably choose and argue based on their values about both mathematics and mathematics education, as well as other values (Seah & Bishop, 2000). Part of this large group of values could be the values as expressed in the three curricula.

Data from one group of each of the three countries were transcribed and analyzed for identifying values in a deductive approach, starting with the existing categories of mathematical and mathematics educational values defined by Bishop and colleagues (see Table 1). Sometimes two or more values were identified in the same extract as we will present in the results section. The first author went through the data from the Norwegian PTs to identify mathematical and mathematics educational values focusing on the descriptions of values by Bishop and colleagues. Since those values could not fully capture the discussions about BMI and broader about mathematics in society, aspects of critical thinking as defined by Jablonka (2020) were identified in this process, resulting from the PTs' engagement with the questions about critical aspects of the BMI and its consequences. An inductive approach, using thematic analysis (Braun & Clarke, 2006) was so used to identify categories of critical thinking about mathematics in society (see Table 2), that were classified and grouped according to their similarities in their focus. There are connections and similarities between the mathematical or mathematics educational values and the mathematics in society values. For example, one of the categories in the mathematics in society, *mathematical and critical knowledge* is connected to the *relevance* as a mathematics educational value. However, there is a distinction between them: while in the latter the focus is on how the mathematics is learned as to solve real problems, the focus on the former is the PTs' emphasis on the relevance of students having both mathematical and critical knowledge, also above school knowledge, to not be fooled. Similar considerations were done during the coding of the data. The second author went through the categorization of the Norwegian[3] (NO) data by the first author and further commented and suggested changes. The same process was used with the South-African (SA) and the Albanian (AL) data. In this process, the initial categories about mathematics in society-critical thinking from the Norwegian data were expanded with new ones based on the South-African and Albanian data. In the end, the categories were further grouped into four definitive categories based on their common focus and were expanded with explanations inside the same category. We had for example initially one category for the uses of mathematics as appropriate or inappropriate, one about certainty and uncertainty of the mathematics in society and something about valid or invalid mathematical models, which was in the end defined as the category *mathematical solutions to social issues* and was expanded to include all the examples from the three groups of data. In the following section the results will be presented.

[3] In the rest of the text we will use NO for Norwegian students, SA for South African students and AL for Albanian students.

Table 1
Mathematical values and mathematics educational values.

Type of values	Dimension	Complementary values at different dimensions	
Mathematical	*Ideological*	*Rationalism*- deductive reasoning, argument and logical thinking	*Objectism*- symbolizing, representations, treating abstract ideas as objects
	Sentimental	*Control*- power and certainty, mastering rules of mathematics	*Progress*- abstracting, generalizing, growth through questioning
	Sociological	*Openness*- for anyone, freedom to choose and explain mathematical ideas, transparence and critique	*Mystery*- of mathematics, fascination over mathematical ideas,
Mathematics educational	*Mathematics learning*	*Formalistic*- receptive mathematics learning, based on deductive reasoning	*Activistic*- inductive reasoning and discovery learning
	Mathematics understanding	*Instrumental*- knowing the rules	*Relational*- knowing why
	Nature of mathematics learning	*Relevant*- solve daily problems, contexts	*Theoretical*- knowledge without contexts
	Mathematics for whom	*Accessibility*- mathematics for all	*Specialism*- mathematics for an elite group
	Applying mathematical knowledge	*Evaluating*- unknown answers, using routine procedures, investigate	*Reasoning*- supplement capability to reason and communicate ideas

Table 2
Categories of critical thinking.

Values	Dimension	Complementary pairs at different dimensions	
Mathematics in society – critical thinking	*Makers and users of mathematical models*	*Trust* in those who make and use mathematics, acceptance of authorities;	*Mistrust* in those who make and use mathematics, questioning their intentions
	Mathematical and critical knowledge	*Relevance* of mathematical and critical knowledge, also above school level, importance of teaching critical thinking	*Lack of* mathematical and critical knowledge: being fooled, harmed, being uncritical
	Consequences of using mathematics	*Good,* systematizing knowledge, generalizable and useful, compare different groups, mathematics for the general man, useful for experts	*Bad*, misleading, bad consequences for vulnerable people, harming (when used in an instrumental way) for the individual; fixating on numbers; discriminating for some, unethical
	Mathematical solutions to social issues	*Universal, appropriate*, unlimited, valid, neutral, culture-free, common sense, necessary	*Not universal,* one size does not fit all, exceptions, counterexamples; not all variables considered; context dependent, limited value; how is the social issued defined, and for whom; not transparent, invalid when against common sense

4. Results

PTs' exhibited values when discussing the different mathematical aspects of BMI, the social aspects, and its hypothetical use in mathematics teaching. The results section is organized for each group of values identified in the data, starting with mathematical values, mathematics educational values and then expressions of critical thinking connected to the mathematics in society values. Data extracts from each national group of students will be presented to illustrate categories.

4.1. Mathematical values

As stated above, a deductive analysis approach was followed for mathematical and mathematics educational values. To recall the mathematical value of *objectism* one is essentially concerned with the symbolization, the meanings of the variables and, in this case how they are used in the BMI. The BMI, mass (kg) divided by the square of the height (m) of a person, was treated as an object, a formula, from which to get a result. Across all three sites the PTs engaged with this object. This happened in the beginning of the discussions, when working with the mathematical aspects of the formula and what it measures. By uttering "they take your height into consideration. They take your mass into consideration and based on that they come up with a number to say if you're healthy or not" a South African PT, for example, merely repeated, in words, the construction of the BMI and linking it to the use for determining the well-being of a person. As another representation of the BMI, students had at their disposal a graph for categorizing people from under-weight to normal to obese. This object was also interrogated as in the excerpt from Norwegian PTs "I might have been a centimeter [taller] and then I would end up there (pointing to a category on the graph), and then over here", which is then précised by another PT "so you had become obese during the day".

Rationalism as deductive reasoning, argument and logical thinking was also present in the discussions of the participants. It was identified when PTs were comparing different formulas that could be used to measure obesity and recalling previous knowledge about exponents and functions "what is your height, so a few centimeters in the second degree can have very big impact" (NO). Similarly, the Albanian PTs were comparing the original formula of the BMI with a hypothetical one, mass divided by height: "For small fluctuations of the height, the value [m/h] changes a lot maybe, while it changes less when it is in the second degree [m/*hxh*]", and so finding arguments for the extant formula.

The PTs went further to question the mathematical aspects of the formula or measurement issues of for example the height as a variable such as “I know that you can have quite a big difference in height if you measure yourself during the day, [or] in the evening…” (NO). Pointing out weaknesses of the formula, as in the extract, by finding counterexamples was categorized as showing the value of *progress* as defined in Table 1. The PTs went further to propose other models to eliminate the weaknesses of the extant formula and discussing them openly in their groups. They took the freedom to choose and explain mathematical ideas and put them forward to critique, showing the value of *openness.* In the following excerpt, a group of South African PTs suggested adding heart rate as a third variable that would affect the BMI intervals so that the sportsperson in the task sheet picture would not be considered obese “So, say you run for two minutes and then your heart rate is 74, say 80 to make it easy. Then they could give, depending on how much you want the scale to move, they could have the heart rate over 3 or 2, so it will be either plus or minus that heart rate. I mean, based on that, that value will be moved, the additional plus or minus where the point lies on this [the chart with BMI accompanying the task sheet]”. Here the PT is showing *openness* by putting forward his ideas of including a third variable in the formula. After the intervention of the lecturer, he came up with the formula “kg/m2 + heart rate/2″, and the lecturer’s remark about different measuring units in the new formula remained unanswered.

It was often so that *progress* was accompanied by *openness* in the data. When PTs questioned aspects of the mathematics, they went on to find other ideas to respond to those issues and being open to scrutiny towards their ideas by other group members or lecturers. Both these values were seen in connection to critical thinking, as premises for one to be critical toward the mathematics in society.

4.2. Mathematics educational values

This second group of values was often seen in connection to the *critical thinking* of the PTs regarding different aspects of the formula and its use. For example, the *instrumental* use of mathematics formulas (BMI and other) was pinpointed as problematic when PTs discussed the BMI’s consequences. “You must have that previous knowledge of what it means.” (NO), referring to the meaning of the BMI number and how it is calculated otherwise “it can go bad”. This latter pointed to the bad consequence of using BMI for people who can get fixated to make changes in their lives only based on the number.

Given the task in the question sheet, the nature of mathematics learning with this activity was placed in the *relevance* value of mathematics to solve societal problems. Probably because of this premise, there not many discussions about the nature of mathematics learning, as theoretical or as relevant for solving problems. On one occasion the South-African PTs showed thoughts about an *activistic* learning of mathematics when discussing possible ways of working with indexes in schools. They talked about “allow[ing] the learners to scrutinize that index.to say but why didn’t they consider these factors” like the way the PTs themselves scrutinized the BMI.

In general, the PTs approached the *accessibility* value when focusing on the importance of persons knowing the mathematics behind the index to make well informed decisions when using such formulas in their lives. One occasion of *specialism* in terms of mathematics models being made by an elite group, was identified when Albanian PTs were discussing changes in the BMI formula or its cut-offs. One of the PTs then added laughing “I don’t know then…if it was [so easy], I would have made a formula myself”.

The PTs in the study used mathematics to *evaluating* unknown answers and investigate when asked to think about other obesity measures, such as m/h, and compare them to the BMI as the Albanian PTs in a previous extract (see 4.1). There were also occasions of PTs using mathematics to supplement *reasoning* as when the Norwegian PTs were trying to argue for the need for a BMI formula for each gender by bringing different numerical values into the formula. One PT started with comparing a girl with 167 cm height and weighing 63–64 kilos, with a 180 cm boy, followed by another PT “a boy who is 175 [cm] and a girl is 175 [cm] will have automatically…totally different…men have more muscles than girls…it is very strange to set them up against each other”.

4.3. Mathematics in society values

We identified four categories consisting of complementary pairs (Table 2) in the PTs discussions, pointing to four aspects of being critical to the mathematics in society. In general PTs expressed *mistrust* towards those who make the indexes as in “So when we look at indexes, I think what we need to consider in society is who is picking those indices. Who is picking the factors?” (SA). This reflection came after the PTs had discussed some disagreements with the BMI index in terms of what it purports to measure. An example showing *trust* in those who make the mathematics is the one used earlier by Albanian PTs expressing the value of *specialism* (see 4.2.).

The earlier discussed value of *instrumental* use of mathematics (see 4.2.) came to the fore here when PTs discussed the *lack of* mathematical and critical knowledge as resulting in *bad* consequences “So it is probably many who take it [the BMI test] and [think] wow I am a little overweight. now i have to get away [and do something about it]. It may go wrong” (NO). Vulnerable people getting obsessed about lowering the BMI were used as examples.

The South-African PTs pointed out several examples to illustrate the problems around applying mathematical solutions to social issues, and those examples were categorized as belonging to the mathematical solutions being *not universal.* Having first questioned the people behind the use of BMI and showing *mistrust*, as in “who says it’s obese to be that?”, they talked about BMI as *not universal* since it does not take account of some variables or contexts such as culture. In the following example, the PTs were referring to one of the indigenous groups in South-African society, and according to the curriculum, including indigenous knowledge is one of the desired values. “So, you’re saying that in your culture if someone is big …fat…you’re saying that that is a sign of wealth?”, one PT asked another one coming from another culture in South Africa. In this kind of discussion, the concept of obesity and its measurement, were problematized as being culture and context dependent, and the formula is seen as deficient since it does not take those into account: “And if this is a westernized formula.it’s totally different”. After such discussions, the South-African PTs investigated the *relevance* of teaching *critical thinking* in mathematics “They [students] just need to learn that someone can alter the picture of what you see….to critically think" and on ideas on how to do this in classrooms.

Judging by the *bad* consequences, the Albanian PTs wouldn't use BMI in school because "it categorizes students, for example you are overweight". Earlier the PTs talked about the use of such an index by their physical education teacher in school and using humor they pointed to such *bad* consequences "the teacher making us obsessed since early age", but also about the *good* consequence of being able to compare different groups or generalizable data.

5. Discussion and conclusion

In this study we gave the PTs in three countries a task to engage them in critical discussions about the role of mathematical models in society, and we focused on the kinds of values that PTs express in their discussions. The values that PTs hold will frame their decisions on how to pursue mathematics teaching and consequently on involving, or not, students in critical discussions about the role of mathematics in society. From a critical mathematics education perspective (Skovsmose, 2016) it is important to know what underlying values can determine if students, or PTs, will engage with the role of mathematics in society, and try to support the development of such values. This is in turn valuable for teachers and teacher educators who want to engage their students with critical discussions about mathematics in society, and in this way develop their critical thinking in mathematics (Jablonka, 2020).

When looking at Table 1, there are some values which are clearly related to critical thinking. *Progress* and *openness* are necessary for one to be critical, thus seeing models and as open to critique and questioning methodological standards of models in terms of Jablonka (2020) and Skovsmose (2016), as is the use of mathematics to supplement *reasoning*. *Openness* seems to be a premise for *critical thinking*, since it opens for criticism of different sides of using mathematics in society, as it was earlier identified in the categories. *Openness* goes hand in hand with the *progress* value where the general formula (BMI) for all kinds of groups of people or different mathematical models were questioned. Also thinking that mathematics is *accessible* to all and not just for an elite, is a premise for undertaking criticism towards mathematics. All these values were also found on the Dede et al. (2021) study of modeling tasks in a socio-critical perspective.

There are nevertheless other aspects that come to the fore when the focus is on *mathematics in society* as in the BMI task, which are not necessarily found in other occasions. This was also the case with the Dede et al. (2021) study where social justice values and the well-being of others were only found in socio-critical modeling tasks. As both Seah and Bishop (2000) and Ernest (2016) also emphasized, the values outside the mathematics classrooms are influenced by, and interconnected with, those inside. In our data, in addition to the mathematical and mathematics educational values (Bishop, 1988; Seah & Bishop, 2000), elements of critical thinking were as well identified. The four categories represent different aspects of the *mathematics in society*: the *people behind* the formulas and their intentions, the *consequences* of mathematical models for different people, the *relevance* of citizens having *mathematical and critical* knowledge to judge mathematical models and questioning the need for and appropriateness of *mathematical solutions* to certain social issues. As Skovsmose (2014) argued, working with extant mathematical models is one way to investigate mathematics in action. The four categories of mathematics in society point to the different aspects of mathematics in action (Yasukawa et al., 2016) which the BMI task and the toolkit afforded. In the PTs' discussions, they pointed to the BMI formula as used to describe the obesity phenomenon, and to one positive aspect of the *consequences of using mathematics* to systematize knowledge, to compare different groups and being useful for experts. They on the other hand pointed to the inscription function of mathematics in action as when they criticized the fact that the BMI formula, that is being used as a lens to see what an obesity is, only consists of two variables, height and weight, problematizing the model's universality in the category of appropriateness of *mathematical solutions to social issues.* This category further corresponds to the prescription function questioned by the PTs in that the BMI is the "tool for calculating what to do" (Yasukawa et al., 2016, p. 83), and thus being given a universality which is not justified; the prescription function of mathematics in action is also visible in the *consequences* category since BMI is the tool being used to prescribe measures to fight obesity, as Hall and Barwell (2015) also argued for, and where the PTs again criticized the measures taken at an individual level only based on the BMI results as being misleading and harming for them. The same category of the values can be connected here to the subscription function of the mathematics in action, as the results from the BMI formula might get accepted uncritically by individuals and this has consequences especially for vulnerable people who might get fixated on the numbers. When PTs emphasized the *relevance* of mathematical and critical knowledge to not accept mathematical results uncritically, they were again exploring the subscription function and pointing to the formatting power of the mathematics (Skovsmose, 1994) and the use of mathematics to value objects as in "the accountant's balance sheet" (Ernest, 2016, p. 111). In addition, the category *the people behind the formula* points to the prescription function as the PTs displayed mistrust in the people authorized to use the BMI in making decisions that have consequences for other people.

Questioning the intentions of using the mathematics formula, its failure to include relevant variables and its validity in different contexts correspond to the *critical thinking* as defined by Jablonka (2020) and help to nuance it. This is in line with Skovsmose's (2014) consideration of mathematical models as re-presentations of reality and Barbosa's (2006) consideration of models as not being neutral descriptions of reality. As discussed earlier, the PTs from all the three national groups did clearly not judge BMI as the appropriate model of obesity in individuals, thus not accepting its universality in measuring obesity (see Table 2) and questioned its appropriateness in different contexts. However, while the Norwegian and Albanian PTs pointed to variables not considered by the formula such as muscles and fat, or gender, the South African PTs were, in addition to those variables, especially skeptical to the formula as being westernized, as it was developed based on data from Europe. All these aspects of the BMI formula and its connection to critical mathematics education were pinpointed by Hall and Barwell (2015) as fruitful for the students to develop critical arguments and become aware of the role of mathematics in society's perceptions of the phenomenon of obesity. Therefore, the toolkit used in the study, opened possibilities for the PTs to reflect about the role of mathematics in society, or what Barbosa (2006) calls reflexive discussions. Further, the South African PTs because of their skepticism on using a formula created in a totally different context than their own, they clearly emphasized the relevance of teaching critical thinking to students, not just in terms of mathematical knowledge, but also in terms of warning students that other people would try to influence them using mathematics. That is, they want to point their students' attention to the role of mathematics in students' lives and society in general, to it not being neutral as many might think, in line with the principles of critical mathematics education (Skovsmose, 2016). While we cannot claim that this is a direct consequence of the PTs engaging with the BMI task, neither that the PTs will change their values, it is certain that the task created the conditions for the PTs to think about and discuss some of the issues connected to mathematics in society and its connection to the classroom mathematics. We find support in Mamolo's (2018) study with PTs who gained awareness about the relevance of using social justice tasks in mathematics to develop students' critical thinking about the world after some sessions where they themselves worked with social justice tasks as learners.

Seah et al. (2016) in discussing a mathematical curriculum based on values, pointed to determining how different mathematical classroom activities can activate different mathematical values as a means to develop values together with the mathematical concepts. The whole toolkit used in the study, opened for PTs to experience, and express many of the complementary pairs of values, similarly to the Dede et al. (2021) study of socio-critical modeling tasks. This is in line with Bishop (1988) who recommended that students should be exposed to all complementary pairs in a balanced way. It also becomes clear from our results that the mathematical values of *openness, progress* and *rationalism* figured often together in the deliberations of the PTs. They were often accompanied with mathematics educational values such as *activism* and *accessibility*, while PTs questioned the intentions of the *people using mathematics* in society and its *consequences*, as well as the *universality* of applying mathematical solutions to social issues. These combinations might be explained with the nature of mathematical modeling, and specifically socio-critical modeling, that opens for connections between social issues and mathematics and being different from the traditional mathematics education, as also Dede et al. (2021) discussed.

We do not claim to have captured the PTs' values in a 60-minute session, and the study was not originally designed around the value concept. However, from a research point of view, the study directs attention to mathematical and mathematics educational values underpinning participants' choices that are imperative in decisions to engage with mathematics in society in critical ways. The novel contribution are the values as connected to being critical to mathematics in society, that emerge when exploring the mathematics that underpin social issues, and their connection to the two other groups of values.

Declaration of interest

None.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jmathb.2023.101035.

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