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The Variation of University Physics Students' Experience of Plus and Minus Signs in 1D Vector-kinematics Revisited

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This article revisits and expands upon a previous phenomenographic study characterising the qualitatively different ways in which South African undergraduate physics students may experience the use of +/– signs in one-dimensional kinematics (1DK). We find the original categorisation as applicable for interpreting Swedish university-level students' responses to 1DK questions. However, by way of a typology of potential learning outcomes associated with +/– signs in 1DK, our review of the topic reveals that the original study's treatment misses the implications of +/– signs related to time rate of change and graphical shape. We also add to the description of students' experience of +/– signs in 1DK by incorporating ideas from the Variation Theory of Learning and by focusing on some of the aspects of +/– signs in 1DK that were underemphasized in the original study. Our analysis thus provides a template for physics educators to support students' conceptual understanding of sign conventions in vector kinematics.

Keywords: *Introductory-level physics; variation theory of learning; patterns of variation; algebraic signs; disciplinary-relevant aspects*

Introduction

The use of mathematics in physics has long been shown to be challenging for introductory-level physics students (Aguirre, 1988; Aguirre & Rankin, 1989; Knight, 1995; Lueck, 1934; Viennot, 1981), not least because of the specialised ways that physicists use mathematical symbols (Brahmia et al., 2020; Redish, 2021; Redish & Kuo, 2015). For example, recently, the use of +/– signs in physics has been identified as troublesome for students, with some scholars identifying 'reasoning about signs' as a central facet of quantitative reasoning in physics (Brahmia et al., 2021).

In this paper, we explore physics students' experience of +/– signs in the context of one-dimensional kinematics (1DK) by revisiting and expanding upon a previous study by Govender (1999, 2007). Govender identified five qualitatively different ways that students may experience +/– signs in the context of 1DK through a phenomenographic analysis (Marton, 1981; Marton & Booth, 1997) of interviews with 19 South African first-year physics student teachers. He found that the variation in how students may experience +/– signs was that they should (A) *not* be applied in 1DK, (B) be applied as *magnitude* only, (C) be applied as *changing magnitude*, (D) be applied as both *magnitude and direction* and/or (E) be applied as *direction*. With new data from Swedish physics students at the level of introductory university physics, we now examine how useful Govender's five categories have remained for describing the variation in how students may experience +/– signs in 1DK and discuss what these categories can imply for the teaching and learning of kinematics. The aim of this paper is, thus, twofold: first, to investigate the applicability of the five categories originally identified by Govender; and second, to expand upon those results to provide recommendations for how to address students' experience of +/– signs in 1DK.

Since phenomenographic analyses are generally not predicated on the positivist notion of reproducibility (Sandbergh, 1997; Sin, 2010), we emphasise that our study is not as much a replication of Govender's (1999, 2007) research as an attempt to revisit Govender's findings vis-à-vis an updated literature base and new data in a different educational context. Relevant to our motivation for reconsidering this topic is the expansion of phenomenography into the Variation Theory of Learning (Marton, 2015; Marton & Tsui, 2004) in the years since Govender's original study.

Literature Review

Govender (1999, 2007) was specifically interested in whether the students in his study correctly experienced +/– signs as implying the direction of kinematics vectors in 1DK. He arranged the five categories of student experience (from A to E) in increasing order of how readily students acknowledged that +/– signs encode the directionality of kinematics vectors. However, +/– signs often indicate a variety of things across and within various physics contexts (Brahmia et al., 2020; Olsho et al., 2021), and as we will elaborate on below, can imply other important details of the disciplinary physics associated with 1DK than merely vector direction. We identify a typology of four disciplinary-relevant aspects (DRAs; see Fredlund, Airey et al., 2015; Fredlund, Linder et al., 2015), namely **DRA_0–DRA_3**, associated with the use of +/– signs in 1DK that make up a more complete appreciation of the topic (Table 1). These DRAs correspond to the potential learning targets for this topic as seen from the discipline. One important characteristic to note from the DRAs mentioned in Table 1 is that not only do +/– signs imply more than the direction of 1DK vectors, but also that +/– signs have different meanings depending on their application to position, velocity or acceleration.

Before discussing **DRA_1–DRA_3**, it is worth noting that, when dealing with kinematics vectors, +/– signs are only understandable in reference to a coordinate system of choice. Choosing such a

Table 1. Typology of the DRAs associated with the use of +/– signs in 1DK.

	In relation to:		
DRA	Position ($\vec{r}$)	Velocity ($\vec{v}$)	Acceleration ($\vec{a}$)
DRA_0 (*coordinate systems*)	+/– signs in 1DK are only understandable in terms of a chosen coordinate system, which is moveable under 180° rotation and/or translation		
DRA_1 (*vector orientation*)	+/– signs tell you if $\vec{r}$ is oriented towards +∞ or –∞ of the coordinate axis, *starting from the origin*	+/– signs tell you if $\vec{v}$ and $\vec{a}$ are oriented towards +∞ or –∞ of the chosen coordinate axis	
DRA_2 (*time rate of change*)	—	+/– signs tell you if $\vec{r}$ is changing towards +∞ or –∞ of the chosen coordinate system	+/– signs tell you if $\vec{v}$ is changing towards +∞ or –∞ of the chosen coordinate system
DRA_3 (*graphical shape*)	+/– signs tell you in which quadrant(s) the *r*–*t* graph resides	+/– signs tell you in which quadrant(s) the *v*–*t* graph resides; and tell you the slope of the *r*–*t* graph	+/– signs tell you in which quadrant(s) the *a*–*t* graph resides; tell you the slope of the *v*–*t* graph; and tell you the concavity of the *r*–*t* graph

coordinate system involves acknowledging that coordinate systems are moveable in the first place (Volkwyn, Gregorcic et al., 2020). Since this notion underpins all the other DRAs, we refer to this aspect as $\mathbf{DRA_0}$.

Disciplinary-relevant Aspect 1 (DRA_1): +/– Signs Imply Vector Direction

$\mathbf{DRA_1}$ is the aspect of +/– signs originally emphasised by Govender (1999, 2007) in ranking his five categories. Since the other two dimensions are zero, in 1DK, position ($\vec{r}$), velocity ($\vec{v}$) and acceleration ($\vec{a}$) can only be oriented in one of two directions. Hence, an important feature of 1DK is that the directionality of vectors can be 'algebraically coded' (Rebmann & Viennot, 1994) solely via +/– signs in relation to a chosen coordinate axis. There is evidence to suggest that students' correct use of +/– signs in physics may hinge on vector-based reasoning as consistent with $\mathbf{DRA_1}$. In a survey of university students from a calculus-based introductory physics course, Brahmia (2018) found that those who used vector-based reasoning in their application of the minus sign were more successful in using the sign in a flexible manner across multiple physics contexts.

Disciplinary-relevant Aspect 2 (DRA_2): +/– Signs can Imply Changing Magnitude

This aspect of +/– signs in 1DK has to do with translating between $\vec{r}$, $\vec{v}$ and $\vec{a}$. Owing to the derivative relationship between these kinematics variables, +/– signs on $\vec{v}$ and $\vec{a}$ imply positive and negative changes in the magnitude of $\vec{r}$ and $\vec{v}$, respectively. Research suggests that +/– signs are often interpreted as meaning increasing or decreasing size in the context of 1DK, although not in the specific way that $\mathbf{DRA_2}$ requires. For example, Ceuppens et al. (2019) found that lower secondary school students working with 1DK often interpreted $-\vec{v}$ as an object 'slowing down' irrespective of the coordinate system. Tabachnick et al. (2018) similarly found that teachers often reverted to a so-called *speed-model* for determining the sign for $\vec{a}$, where $+\vec{a}$ meant 'speeding up' regardless of the direction of $\vec{v}$ (see also, Kranich et al., 2015). These findings resemble those of Dall'Alba et al. (1993), who found that 'positive is speeding up' and 'negative is slowing down' heuristics were indiscriminately applied in students' answers relating to $\vec{a}$ in 1DK. Since there are instances where such heuristics hold true, it is important to emphasise that these mistakes identified by the above studies can be seen as errors or misapplication rather than entirely incorrect.

Disciplinary-relevant Aspect 3 (DRA_3): +/– Signs Imply Graphical Shape

The final aspect of +/– signs that we highlight relates to graphical shape. +/– signs entail important qualitative differences when representing 1D motion via $\vec{r}$ vs. time, $\vec{v}$ vs. time, or $\vec{a}$ vs. time graphs. As Volkwyn, Airey et al. (2020) discuss, +/– signs in 1DK dictate the 'generic shape' (the slope and concavity) of the graph as well as in which quadrant the graph resides (see also Ceuppens et al., 2019).

The Variation Theory of Learning

Having reviewed some of the core ideas associated with +/– signs in 1DK, we now discuss a relevant theoretical innovation that has occurred in relation to phenomenography. Since Govender's (1999) original thesis work on +/– signs in 1DK, the foundations of phenomenography have been expanded upon into the Variation Theory of Learning (VTL) (Marton, 2015; Marton & Tsui, 2004). A core tenant of VTL is that the necessary condition for learning is that students experience purposeful variation of the DRAs for the concept at hand. From the perspective of VTL, variation of DRAs opens the way for a learner to notice those aspects in a way that they would not have been able to before (for more detail, see Marton & Booth, 1997; Marton, 2015; Marton & Pang, 2013; Marton & Tsui, 2004).

In their work connecting phenomenography to VTL, Marton and Pong (2005) highlight how the categories identified in phenomenographic analyses can be described via two criteria: that is, (1) in terms of what is explicitly referred to within each category (the *referential aspect*) and (2) in terms of what is implicitly in focus within each category (the *structural aspect*). As was typical of phenomenographic work at the time, Govender (1999, 2007) only presented the referential aspect of each of the five categories he identified. That is, he described each category in terms of how the students in his study described using (or not using) +/– signs in 1DK. In our re-examination of Govender's work for the present paper, we add an account of the *structural aspects* of Govender's categories as well—highlighting the features of +/– signs in 1DK that are in focus implicitly within student responses for each category. Beyond making the categories more understandable, detailing the structural aspects of each category should support physics education researchers and physics teachers in recognising what the categories imply for the teaching and learning of physics.

VTL has been applied in a variety of different research contexts as a means of interpreting learning scenarios (e.g. Euler et al., 2020; Ingerman et al., 2007; Kullberg et al., 2016) and of structuring teaching sequences through so-called *patterns of variation* (e.g. Attorps et al., 2016; Thuné & Eckerdal, 2009). With one of our aims being to elaborate on how teachers can change their instructional practices based on Govender's categories, we make use of patterns of variation from VTL for the context of +/– signs in 1DK. The patterns of variation discussed in Marton (2015) are *contrast*, *generalisation* and *fusion*. Marton (2015) asserts that these patterns of variation should occur in a certain sequence for maximal learning to take place; first contrast, followed by generalisation and then fusion. In *contrast*, the learner experiences variation of the focused aspect while the other aspects of the phenomenon are kept invariant. This increases the likelihood of the learner 'picking out' (noticing) the aspect in question. In *generalisation*, the focused aspect is kept constant while another, related aspect is varied. As the name implies, this is intended to help the learner generalise the focused aspect across several surrounding contexts. In *fusion*, both the focused aspect and the unfocused aspects are varied simultaneously in order to show the learner how those aspects are interrelated within the given phenomenon. We will return to these patterns of variation in our discussion of the implications of our analysis for teaching.

Methodology

Sixty Swedish prospective natural science university students were issued 1DK problem-solving questionnaires. These students participated in a natural science preparatory programme intended to prepare them for a natural science degree at university. This means that they had completed upper-secondary school but had not yet completed the required courses in at least one natural science subject to be accepted to a university science or engineering programme. The level of physics taught within this natural science preparatory programme is essentially equivalent to that in a university-level introductory algebra-based physics course. Following an initial analysis of student responses, five of the students were selected for semi-structured interviews. Participation was on a voluntary basis and all participating students had been exposed to algebra-based vector kinematics. Both the questionnaire and interviews were conducted in Swedish to ensure that the students were able to express themselves as accurately as possible.

Questionnaire Design

The questionnaire, inspired by the format of the *Tutorials in Introductory Physics* (McDermott et al., 1998), was originally designed in English by the lead author who is proficient in both English and Swedish. Prior to data collection, the questionnaire was piloted and refined three times with other physics education researchers and physics students. Ultimately, the team of researchers reached a consensus on the questions' effectiveness for probing students' experience of +/– signs. When the final English version of the questionnaire had been agreed upon, it was translated into Swedish, with a back-translation into English undertaken to ensure that no shifts in meaning had arisen from the translation. The questionnaire (see Appendix A) comprised two conceptual 1DK problems explicitly prompting students to explain how they used +/– signs. The first problem involved a ball rolling horizontally on a frictionless surface, while the second problem involved the side-by-side motion of two cars. Both problems involved linear motion in opposite directions.

Interviews

Following an initial analysis of the questionnaire responses, five of the students were selected to take part in stimulated recall interviews such that a variety of students' experience was captured. The five students were selected as a purposeful sample (Patton, 1990), with the aim of the interviews being to clarify how those five students had selected their answers on the questionnaire and to obtain further insights into how they experienced the use of +/– signs when solving 1DK problems. The interviews followed a semi-structured format, where the students' responses to the questionnaire provided a starting point for open-ended discussions about the use of +/– signs in 1DK. Follow-up questions included, for example, 'Can you explain why you used [a +/–] sign for velocity in this problem?' Students were encouraged to elaborate on their thoughts and each interview lasted roughly 15 minutes. All interviews were audio-recorded as per the students' informed consent and transcribed verbatim for use in the analysis.

Analytic Approach

We used Govender's (1999, 2007) original categories as a guide for interpreting both the questionnaire and interview answers. By using a set of existing categories—in this case, from Govender's (1999, 2007) original phenomenographic study—it is important to emphasise that we are not conducting a phenomenographic study ourselves. In this way, our analytical approach could be described as an evaluative, second cycle coding (Saldaña, 2015). An initial analysis was done by the lead author in Swedish to ensure that any nuanced, language-specific meanings were retained. Once all the student responses had been categorised, a sample of the data was given to the other researchers involved in the study to check the categorisation. It was in this second stage of the analysis that we developed the typology of DRAs associated with the use of +/– signs in 1DK (Table 1). The typology was useful in going beyond Govender's original emphasis of how +/– signs encode the directionality of kinematics vectors. In all stages of the analysis, special attention was paid to the potential need for any additional categories beyond those identified by Govender.

Results

From our analysis of the questionnaire and interview data we identified four of Govender's original five categories as relevant descriptors for students' experience of +/– signs in 1DK (namely, Categories A, C, D and E). To reiterate, the categories identified by Govender (1999, 2007) were that +/– signs should (A) *not* be applied in 1DK, (B) be applied as *magnitude* only, (C) be applied as *changing magnitude*, (D) be applied as both *magnitude and direction* and/or (E) be applied as *direction*. Through our analysis we were not able to identify any additional category beyond those identified by Govender. It is worth noting that Govender identified several subcategories of experience in his original study, which are more specific than these five major categories. However, for the purposes of our analysis, we assert that the five main categories provided a sufficient level of detail.

Govender's Category B, which we did not use to describe any student responses in our data, includes students' assertions that +/– signs imply positive or negative integer numbers on a number line (i.e. involving a coordinate axis but denying the vector nature of kinematics variables). The absence of Category B in our data could suggest that the framing of our questions was different enough from those used in Govender's study to elicit a qualitatively different subset of student experiences. However, phenomenographic analyses like those conducted by Govender aim to map the *possible* categories of student experiences, not the categories that will *necessarily* appear in every cohort of students. Regardless, we assert that the absence of Category B in our data is theoretically unproblematic.

In what follows, we review Govender's categories that we found relevant for the analysis of our questionnaire and interview data. For each category, we review the meaning students ascribed to +/– signs for the category (i.e. the referential aspect). Then, in going beyond the category descriptions originally offered by Govender, we discuss the structural aspects of +/– signs in focus for experiences within each category as well as how each category relates to the DRAs of +/– signs in 1DK (Table 1). Of the categories we did find useful for describing our data, Govender argued that Categories A, C and D represent inappropriate ways of experiencing the usage of +/– signs in 1DK, while Category E represents a disciplinary appropriate experience. We will point out not only the correct things as part of the responses in Categories A, C, and D, but also the things that responses in Category E leave out. All data incorporated below are drawn from the completed questionnaires and/or the interviews.

Category A: +/– Signs are not Applicable to 1DK

This category includes student responses that describe +/– signs as not being relevant to 1DK. One student, for example, asserted the following:

> S59: You cannot use arrows or signs to describe the velocity or acceleration, only numbers. I have at least never encountered anything other than numbers to describe this within physics.

An answer like S59's implies that such a student may have little appreciation of the utility of 'symbolic representations in [1DK]' (Govender, 2007: 66). The structural aspect of this type of response is that there is *no focus* on +/– signs. It may be that students who describe their experience of +/– signs in this way have some working knowledge of how to solve problems in 1DK, but at least when directly asked to consider the role of +/– signs, they downplay the vector nature of kinematics variables entirely. Since this type of response is not apparently 'adjacent' to any of the DRAs of 1DK, teachers might do well to encourage students with this experience to first recognise the importance of directionality in kinematics variables.

However, something that was underemphasised by Govender in his original appraisal of Category A was that some students were technically correct in their understanding of kinematics vectors, but simply expressed a preference for other directional labels than +/– signs:

> S4: I don't really think in terms of plus and minus, but I think in terms of right and left. A motion to the right feels positive and to the left negative. I now realise that I think that plus and minus seem a bit unnecessary. Why don't you just say a motion to the right or left?

Such students may grasp that +/– signs can be used, yet they specify that these signs are not required when dealing with motion in 1DK. For this reason, an alternative structural aspect of Category A is a focus on the *grammatical redundancy* of +/– signs (i.e. that +/– signs are unnecessary alongside directional labels like right, left, up, down, North, South, etc.). It is important to emphasise, however, that while egocentric directional labels like right/left and geocentric labels like North/South can technically work for many 1DK contexts, there are clear advantages to the consistent use of +/– signs (and in choosing a useful coordinate system, as per $\mathbf{DRA_0}$) that students would benefit from noticing. For instance, egocentric coordinates are fixed relative to specific viewpoints and geocentric coordinates are fixed to the orientation of the Earth. Such terminology can contribute to the harmful notion that coordinate systems, however implicit, are fixed beyond the control of the student (see Volkwyn, Gregorcic et al., 2020). Especially since 1D kinematics is often taught as the foundation upon which 2D and 3D kinematics are later built, it is likely to be advantageous for students to notice the disciplinary utility of +/– signs early on. Nonetheless, by implicitly acknowledging the utility of directional labels and that kinematics variables have a directional component, students who describe their experience of +/– signs in a manner similar to S4 could be seen as 'nearby' $\mathbf{DRA_0}$ and $\mathbf{DRA_1}$.

Category C: +/– Signs are Applied as Changing Magnitude

This category includes student responses that describe +/– signs as implying that a kinematics variable is getting bigger or smaller. For example, like in the two quotes below:

> S28: I experience plus as something that is getting bigger, and minus as something that is getting smaller.

> I: What do the signs for acceleration mean to you?
> S6: Increase or decrease of velocity.

This type of response, which was common in our data, shows a potentially correct application of +/– signs in line with $\mathbf{DRA_2}$ (at least for S6). The structural aspect of such responses in Category C is a focus on how +/– signs imply a *time rate of change*. From the excerpt above, however, we do not have enough information to determine if S6 is using incorrect heuristics like those found in previous 1DK studies (Dall'Alba et al., 1993; Kranich et al., 2015; Tabachnick et al., 2018). A more complete appreciation of $\mathbf{DRA_2}$ should acknowledge that a positive $\vec{a}$, for example, entails a change in $\vec{v}$ in the direction of +∞ with respect to the chosen coordinate system—which often implies a *decrease* in the magnitude of a negative $\vec{v}$.

The characterisation of Category C responses is confounded further when we see that some students incorrectly interpret +/– signs as implying a change within the same variable (identified also by Ceuppens et al., 2019):

> I: What do you think that the signs for velocity show?
>
> S27: Plus to me means that it is going faster, that the velocity increases. And minus should then be the opposite, that the velocity simply decreases.

Importantly, students that articulate this experience of +/– signs may only be missing one key detail of $\mathbf{DRA_2}$: that +/– signs used for one variable can be used to tell you about a change in magnitude for *another* variable, but not for the variable on which the sign is applied. Still, a worrying possibility is that students may experience +/– signs as implying a change within the same variable (as with S27) and, thus, may be using incorrect heuristics like '+ *always* implies increase' and '– *always* implies decrease' as mentioned before.

S27's answer here, again, shows some awareness of the use of +/– signs to describe directions but, like S28 and S6, they still interpret these signs to mean a change in magnitude. This may be an indication that these students have noticed how the sign of $\vec{a}$ tells you about the rate of change of $\vec{v}$ ($\mathbf{DRA_2}$), for example, but have not yet simultaneously noticed the directionality aspect of +/– signs with regards to a coordinate system ($\mathbf{DRA_1}$).

Category D: +/– Signs are Applied as Both Change in Magnitude and Direction

This category includes student responses that describe +/– signs as implying both a change in magnitude and vector direction. The structural aspect for responses in Category D is a focus on the time rate of change of kinematics variables *and* orientation of kinematics vectors. At first, this category of experience would appear to be the closest to the disciplinary-correct interpretation of +/– signs in 1DK since a complete appreciation of the topic should simultaneously involve $\mathbf{DRA_1}$ and $\mathbf{DRA_2}$. However, in both our data and the data presented by Govender (1999, 2007), students often switched between interpretations of +/– signs at inappropriate times. For example, the following student changed their interpretation of +/– signs depending on the variable in question:

> S2: When it comes to velocity, plus and minus only show direction. When it comes to acceleration they only show the acceleration's increase or decrease and don't take direction into consideration. Why it turned out this way I don't know!

In this response, S2 indicates that there is something inherently contradictory or inconsistent about such a use of +/– signs, but asserts that signs on $\vec{v}$ and $\vec{a}$ should be interpreted differently. There are seeds of the correct reasoning present in such an answer: +/– signs do imply the direction of $\vec{v}$ (as per $\mathbf{DRA_1}$) and do, in fact, imply something different when they are applied to $\vec{v}$ and $\vec{a}$ (as per $\mathbf{DRA_2}$). However, there are obvious issues with such partially correct interpretations of +/– signs in 1DK. For S2, there is also the mistake seen in Category C answers of judging +/– signs as implying change within the variable on which they are applied.

Another student (S8) switched between interpretations of +/– signs depending on what was happening in the physical situation:

I: Would you describe [the acceleration] using plus and minus for this ball?

S8: […] I would describe it as both positive and negative. Positive when it is [released] towards the wall. Negative when it hits the wall and positive when it leaves the wall again.

I: Why was it negative when it hit the wall?

S8: Because it is a deceleration and it becomes negative. The speed will decrease and then the acceleration is negative so to speak.

In this excerpt we see that the student interprets acceleration to be positive in the direction of motion (before and after the collision with the wall), indicating that +/– signs on acceleration are used to describe the direction of observable motion. During the ball's contact with the wall, however, +/– signs are used to describe a decrease in speed. Here again, there are features of $\mathbf{DRA_1}$ and $\mathbf{DRA_2}$ present in S8's answer, but they are not applied consistently.

This inconsistent way of using +/– signs may indicate that learning here involves, among other things, challenges with making connections between +/– signs and the details of the problem context. Marton and Pong (2005) found similar shifts in students' experience in an unrelated context, pointing out that students' experience likely shifted as they shifted their focus within the problem (i.e. as the structural aspect changed for the students). We reiterate here that a disciplinary-correct experience of +/– signs requires precisely this type of shifting between meanings for $\mathbf{DRA_1}$, $\mathbf{DRA_2}$ and $\mathbf{DRA_3}$—students simply may not be aware of how and when to appropriately make such shifts.

Category E: +/– Signs are Applied as Directions

The final identified category includes student responses that describe +/– signs as implying vector directionality. We identify the structural aspect of responses in Category E as a focus on the orientation of kinematics vectors. For example,

S2: [+/– signs describe] the direction, partly. Or the direction in relation to how you choose it.

Govender (2007) identified Category E as the 'correct scientific conception' (p. 65) for +/– signs in 1DK. Here, S2 correctly acknowledges that +/– signs imply direction (as per $\mathbf{DRA_1}$) while also recognising that this directionality is a choice (as per $\mathbf{DRA_0}$). It should be clear, however, that there are other important implications of +/– signs in the context of 1DK, such as those related to $\mathbf{DRA_2}$ and $\mathbf{DRA_3}$. As such, Govender's original praise for Category E responses should be tempered slightly: at best, responses within Category E represent only partially correct conceptions of +/– signs in 1DK.

It is, again, also important to note how the experience of +/– signs may be linked to certain kinematics variables and not others (as seen in Categories C and D). For example, the following excerpt, where a student is discussing the motion of a ball rebounding after an elastic collision, shows +/– signs only explicitly applied for the direction of the ball's motion (i.e. velocity):

I: Why was the motion of the ball positive to start with?

S6: I guess that it is just something I assume or presuppose.

I: And then it becomes negative, why?

S6: Well, after it turns it can't continue to have the same sign as before. I think that it should be negative.

Although such a response could indicate a (partially) correct interpretation of +/– signs, S6's experience of signs should be more explicitly clarified for all the kinematics variables and for the contexts related to $\mathbf{DRA_2}$ and $\mathbf{DRA_3}$.

Summary of Analysis

Table 2 summarises the referential aspects, structural aspects (following Marton & Pong, 2005) and the 'nearby' DRAs of each of Govender's (1999, 2007) categories. By 'nearby' DRAs (listed in the final column of Table 2), we mean those DRAs that are either already consistent with student responses within each category or could be immediately built towards with the guidance of a teacher. Our intention with focusing on these nearby DRAs is to highlight the things that may already be correct or very nearly correct in students' responses within each category. For example, Category C responses often categorically miss the experiences of +/– signs in 1DK that align with $\mathbf{DRA_1}$ and $\mathbf{DRA_3}$, while misapplying aspects of $\mathbf{DRA_2}$. However, Category C responses will be nearby $\mathbf{DRA_2}$ since the structural aspect of that category is a focus on the time rate of change implied by +/– signs. An attentive physics teacher may only need to point out that +/– signs imply a change in magnitude of a different variable.

Still, it is important to reiterate that none of the student responses in our data or in the data presented by Govender emphasised the implications of +/– signs for the shape of kinematics graphs (as per $\mathbf{DRA_3}$). As such, although correctly applied Category D responses would be the closest to a complete appreciation of +/– signs in 1DK of the categories identified in our work, the framing of the questions in our study and Govender's original study were possibly designed in such a way as to emphasise only $\mathbf{DRA_1}$ and $\mathbf{DRA_2}$ contexts. It remains to be seen if a different set of questions that stressed the graphical representation of 1DK would produce one, two or many more categories of experiencing +/– signs. Indeed, physics educators have long encouraged students to work with graphical representations in 1DK (Trowbridge & McDermott, 1980, 1981), and it could be the case that students coordinate the many other experiences of +/– signs around kinematics graphs (Volkwyn, Airey, et al., 2020).

This leads to a related comment about how we should assess student understanding of +/– signs in 1DK. A 'fully correct' interpretation of +/– signs in 1DK, as identified in the DRA typology presented in Table 1, requires focusing on how these signs are used for various purposes: with $\mathbf{DRA_1}$, +/– signs are used for visually orienting vectors with respect to a coordinate axis (pertinent in the setup of a kinematics problem); with $\mathbf{DRA_2}$, +/– signs are used to translate between the different kinematics variables (more pertinent in the 'working out' of a kinematics problem); and with $\mathbf{DRA_3}$, +/– signs are used to generate and interpret kinematics graphs (more pertinent for visually summarising 1D motion). We can expect students to recruit a different experience of +/– signs depending on what part of 1DK is in focus (Marton & Pong, 2005). Items with a single focus will possibly miss prompting students to attend to all of the DRAs of +/– signs in 1DK.

Implications for Teaching

Our results indicate that teachers of 1DK who prompt students with similar problems to those used in our study can expect their students to experience +/– signs in a handful of specific ways. Importantly, the structural aspects of the categories provide an indication of what students will possibly focus on when dealing with +/– signs in 1DK. So, in working with students to help build conceptual understanding of this topic, teachers would do well to encourage attention to

Table 2. Expansion of physics students' experiences of the use of +/– signs in 1DK.

Category	Meaning ascribed to +/– signs (*referential aspect*)	Elements of +/– signs in focus (*structural aspect*)	DRAs 'nearby'
A	+/– signs are not applied	*Not focused* on +/– signs or focused on their *grammatical redundancy*	$\mathbf{DRA_0}$, $\mathbf{DRA_1}$
B	+/– signs are applied as magnitude only	Focused on the *countable size* implied by +/– signs (as with money)	$\mathbf{DRA_0}$
C	+/– signs are applied as changing magnitude	Focused on the *time rate of change* of kinematics variables	$\mathbf{DRA_2}$
D	+/– signs are applied as changing magnitude and directions	Focused (often inconsistently) on time rate of change of kinematics variables *and* orientation of kinematics vectors	$\mathbf{DRA_1}$, $\mathbf{DRA_2}$
E	+/– signs are applied as directions	Focused on the *orientation* of kinematics vectors	$\mathbf{DRA_1}$

aspects of +/– signs corresponding to the desired learning target (as per $\mathbf{DRA_0}$, $\mathbf{DRA_1}$, $\mathbf{DRA_2}$ or $\mathbf{DRA_3}$). As mentioned in our presentation of VTL, a promising teaching approach for encouraging students to attend to certain aspects of a phenomena involves patterns of variation (Lo et al., 2006; Marton, 2015).

While a full account of instructional recommendations for each of the DRAs is beyond the scope of this paper, in what follows, we present an example of how the patterns of variation recommended by VTL—i.e. *contrast*, *generalisation* and *fusion*—could be used to structure a learning sequence around $\mathbf{DRA_2}$ (that +/– signs tell you about the *orientation* of kinematics vectors in 1DK with respect to a chosen coordinate system).

With the *contrast* pattern of variation, noticing how +/– signs denote the direction of a kinematics vector in 1DK could be supported by varying the direction of a kinematics vector while holding all the 'background' aspects constant. For example, contrast around $\mathbf{DRA_2}$ could involve taking a position of invariant magnitude, say 5*m*, and then varying the sign of that position (+5*m* ↔ –5*m*) against the background of an invariant coordinate system. The key here is that, by varying only the sign of position, it is made noticeable for students that +/– signs encode the direction of the position vector, $\vec{r}$ (distinct from magnitude and the coordinate system). Next, with the *generalisation* pattern of variation, the sign of kinematics vectors should be held invariant while either the magnitude or the coordinate system is varied. For example, generalisation around $\mathbf{DRA_2}$ could start with an $\vec{r}$ of +5 *m* and then translate the coordinate system to the left or right (shifting the origin) while maintaining a positive position. Finally, with the *fusion* pattern of variation, multiple aspects should be varied at once to demonstrate their interdependence. For $\mathbf{DRA_2}$, this could involve rotating the coordinate system underneath an $\vec{r}$ of +5*m*. Alongside the generalisation from the previous step, this rotation would highlight the moveability of the chosen coordinate system (relating to $\mathbf{DRA_1}$ of +/– signs in 1DK; see Volkwyn, Gregorcic et al., 2020) while also emphasising that such a transformation of the coordinate system results in changing the sign of the 1D position.

Conclusion

We have revisited and re-examined Govender's (1999, 2007) interpretation of the qualitatively different ways in which students may experience +/– signs in 1DK. Following a distinct data collection methodology implemented within a distinct educational setting, we identified four of Govender's five original categories as relevant descriptors for students' experience of +/– signs. These results speak to the applicability of Govender's original categorisation and to the consistency with which students will come to experience +/– signs in similar 1DK problem contexts.

Nonetheless, it is relevant to expand as we have on Govender's original (referential) description of the five categories in terms of structural descriptions of the categories and in terms of the DRAs of +/– signs in 1DK. We emphasise that the hierarchy of categories originally suggested by Govender misses at least two of the meanings implied by +/– signs in 1DK related to the time rate of change and graphical shape of kinematics variables. Furthermore, Govender's original analysis fails to acknowledge that +/– signs can mean qualitatively different things for position, velocity and acceleration. Beyond being a useful analytical tool for this paper, the typology of DRAs we identified for +/– signs in 1DK is also useful for educators in clarifying the potential learning outcomes for this topic. Our discussion of the DRAs throughout the analysis shows how student responses, such as those found in our data, are often adjacent to some of the key ideas involving +/– signs in 1DK as identified in the existing literature (e.g. Kranich et al., 2015; Tabachnick et al., 2018; Ceuppens et al., 2019). Finally, via the suggested patterns of variation from VTL, we have also provided a template for how educators might work with students to develop conceptual understanding of this topic through purposeful variation.

There are two major limitations to this study, the first related to analytic bias and the second to the absence of a graphical category in students' experiences of +/– signs in 1DK. With respect to analytic bias, it should be acknowledged that we engaged with our data with Govender's categories intentionally in mind. This means that we may have been predisposed to seeing students' responses as consistent with Govender's categories even when a new category could have been needed. Second, the fact that the graphical component of the use of +/– signs, as per **DRA$_3$**, was not identified among any student answer indicates that the questions in our study (and in Govender's) possibly failed to elicit a specific category of experience from students relating to graphs in 1DK. We suggest that this should be the focus of future studies.

Still, our study contributes to the collective understanding of the teaching and learning of vector-kinematics by explicitly documenting the ways in which students experience +/– signs and suggesting future avenues for research and teaching related to how they use +/– signs in 1DK.

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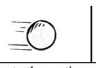
Appendix A

The two problems given in the questionnaire (translated from the original Swedish).

Problem 1: The motion of a rolling ball

A small ball rolls along a smooth surface (ignore friction). When the ball has rolled 2 m, it reverses when it hits a barrier (no energy is lost during the collision) and it rolls back to its original position. For the questions below, please **explain** your reasoning carefully.

Before:  After: 

1.1: Is there any difference to the motion of the ball before and after the turn? Explain the **meaning** of any algebraic signs (+ or –) that you used.

1.2: Is direction important to specify the motion of the ball? Explain.

1.3: Describe the distance and displacement of the ball before and after the turn. Explain the **meaning** of any algebraic signs (+ or –) that you used.

1.4: Describe the speed and velocity of the ball before and after the turn. Explain the **meaning** of any algebraic signs (+ or –) that you used.

1.5: Describe the acceleration of the ball before and after the turn. Explain the **meaning** of any algebraic signs (+ or –) that you used.

Problem 2: Velocity and acceleration of a car chase

Imagine the following sequence:

(1) A police car is standing by the side of the road at the intersection between Dag Hammarskjölds väg and Kungsängsleden when the driver sees a Volvo travelling at a constant speed through a red light.

(2) The police car immediately starts chasing the Volvo, along a straight part of the road, accelerating from rest until reaching a maximum chasing speed.

(3) The officer holds this speed until she is alongside the Volvo.

(4) She turns on the blue light signalling to the Volvo to pull over. The driver of the Volvo starts to slow down, the police car also slows down, staying alongside the Volvo.

(5) Both cars finally stop by the side of the road.

2.1: For the different parts of the sequence (1)–(5) above, **sketch** the velocity of the **police car** using arrows, and signs if appropriate.

(1) (2) (3) (4) (5)

2.2: Explain the **meaning** of **any** algebraic signs (+ or –) that you may have used.

2.3: For the different parts of the sequence (1)–(5) above, **sketch** the acceleration of the **police car** using arrows, and signs if appropriate.

2.4: Explain the **meaning** of **any** algebraic signs (+ or –) that you may have used.

2.5 Suppose the police car **turns around** and follows the **exact same** sequence in the other direction.

2.5.1: For the different parts of the sequence (1)–(5), **sketch** the velocity of the **police car** using arrows, and signs if appropriate.

2.5.2: Explain the **meaning** of **any** algebraic signs (+ or –) that you may have used.

2.5.3: For the different parts of the sequence (1)–(5), **sketch** the acceleration of the **police car** using arrows, and signs if appropriate.

2.5.4: Explain the **meaning** of **any** algebraic signs (+ or –) that you may have used.

2.6 Are there any differences between the arrows and/or signs that you used to describe the velocity and acceleration respectively in the above questions? If so, please explain.